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Division 5, National Defense Research Committee of the Office of Scientific Research and Development

Some Aspects of the Design of Homing Aero-Missiles

by

Hugh L. Dryden
National Bureau of Standards

A report to Division 5 from the National Bureau of Standards

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Approved for National Bureau of Standards by

Lyman J. Briggs, Director.

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Preface

The work described in this report is pertinent to the projects designated by the War Department Liaison officer as AC-1, AC-36, and AC-42 and to the projects designated by the Navy Department Liaison officer as NO-115, NO-174, and NO-235. This work was carried out and reported by National Bureau of Standards under a transfer of funds from OSRD with the cooperation of the Washington Radar Group of the Massachusetts Institute of Technology and Section Reig of the Bureau of Ordnance, Navy Department.

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I. Introduction

Modern warfare is primarily a warfare of missiles - bullets, shells, rockets, bombs, and grenades. The chief problems are to transport, project or propel, and direct these missiles accurately to strike targets of military importance. Missiles may be transported to the vicinity of a target by men on foot, by mobile artillery, by tanks, by aircraft, or by ships. They may be dropped from aircraft, projected from guns on land, ship, or aircraft, accelerated for all or parts of their trajectories by rocket motors, or they may be self-propelled. Most of the missiles in current use are not under the control of the user after launching. Some form of sighting device is necessary to establish the initial direction of travel in a manner to cause the missile to strike the target. Gun sights, bomb sights, and the complicated gun directors and computers are mere aids to determine where to point the gun or release the bomb, and after release of the missile no further control is possible.

For many years much thought has been given to the possibilities of guiding missiles after their release to correct for errors in sighting and for evasive action of moving targets. The development of radio communication stimulated inventive activity in this field and even during the first World War there were experiments on radio-directed aircraft. There are innumerable patent applications relating not only to radio-

The third type, illustrated by the German HS 293 and FX and the Division 5 NDRC Azon and Razon series, uses a remote human pilot who may either see the missile and target visually, aided by flares on the missile and by sighting devices, or be guided by a repeat-back of television or radar information from apparatus contained in the missile, or be guided by radar location of the missile in relation to map location of the target or radar location of the target.

The fourth type, of which there is no existing illustration, uses some type of beam directed toward the target which the missile automatically follows.

The fifth type is the target-seeking or homing type, illustrated by Division 5 NDRC Pelican, Bat, and Felix. Such a missile must utilize some physical property of the military target which causes it to stand out from the background. The most commonly suggested property is the emission or reflection of electromagnetic radiation. Separate techniques are available for transforming three major divisions of the electromagnetic spectrum into directional information. Of the three - visible light, infra red, and radio - radio frequencies, for technical reasons, hold the most promise of useful weapons. Radio techniques as developed in radar are directly applicable. Visible light and infra-red are useful for certain specific types of targets. Other properties have been suggested, for example, the emission or reflection of sound. Investigations along this line for aero-missiles have not given promising results.

Necessity of Technical Coordination.

This report is concerned with various aspects of the design of this last type, the homing aero-missile. The impression is prevalent that scientific advances in many fields have progressed to the point where the development of such a missile is purely a matter of engineering design on the part of the several specialist groups with the usual coordination as to dimensional requirements, weights, and time of completion. Experience has taught otherwise. Optimistic time schedules based on such an assumption can not be met. The development of successful homing aero-missiles requires the solution of certain research problems associated with the complete article involving complex relationships between the performance characteristics of the component parts. There is required a type of overall technical coordination beyond that required in the design of aircraft as ordinarily practiced.

The required technical coordination is made difficult by the wide variety of specialists of different scientific and technical background and accustomed to different vocabularies and habits of thought whose work must be coordinated. For example, in the case of a propelled radar homing missile there will be represented experts in aerodynamics, aircraft structures, propulsion, servomechanisms, electronics, radar, computers, explosives, and fuses. Other types of missiles will require experts in radio, optics, infra-red radiation, heat, etc. No one person can be expert in all of these diverse fields, but the success of the project requires a project engineer who has sufficient knowledge of these fields to be able with the help of advice from the specialists to assume technical leadership in the solution of research problems associated with the system as a whole.

In the design of any homing missile, there soon emerge a number of problems which cut across the boundaries of the specialist groups. The particular design which seems best to one group of specialists creates difficult problems for other groups, and the requirements put forth by the several groups as optimum are often contradictory. For example, certain errors are introduced unless the intelligence device "looks" along the direction of motion, i.e. is accurately bore-sighted. The conventional airplane flies at an angle of attack which varies with the position of the elevator. If the aerodynamics specialist adopts a conventional aircraft design with elevator control, the intelligence device must be coupled to the elevator control in such a manner as to compensate for variations of angle of attack for all conditions of flight. However, the designer of the intelligence device might properly suggest that the aerodynamicist design a vehicle which does not change its attitude with application of the controls. It becomes a matter of research to determine which solution gives greater accuracy and hence how responsibility is to be allocated between the two specialists.

Purpose and Background of this Report.

These broad aspects of the design of homing aero-missiles are treated in the present report. An attempt has been made to make the discussion general in character and applicable to all such missiles, whether propelled or not. It should be stated, however, that the discussion arises from experience with the radar homing missiles of the Pelican and Bat series, and this account undoubtedly reflects the solutions there adopted, as well as the problems peculiar to radar homing.

II. Target Discrimination and Tracking

Limitations of Mechanisms as Compared to Human Brain.

In visual shooting or bombing, the target is identified by the pattern of optical radiation as perceived by the human eye and interpreted by the human brain. In radar fire control or bombing, the target is identified by the pattern of short-wave electromagnetic radiation exhibited on the screen of a cathode ray tube as perceived by the human eye and interpreted by the human brain. During the flight of a homing aero-missile these radiation patterns must be made to operate control mechanisms and the element of interpretation by a human brain is absent. This introduces many problems and severe limitations. A mechanism can perceive only a limited number of physical characteristics of the target pattern and can exercise no judgment

single large discontinuity within the field or on such a path that a single target comes within the field and depends on the precision of the controls of the missile to keep the missile tracking this target.

In order to obtain information about the location of the target, a homing missile must be directionally sensitive. Usually the received intensity of radiation is greatest when the axis of the radiation-receiver is pointed directly at the target. Thus Fig. 1 shows a portion of the response curve of the receiving antenna of the radar receiver used in the Pelican project. The relative power is plotted in terms of decibels, a logarithmic unit, but the power ratios are also indicated. The width at 1/2 power is 23° and the width at 1/10th power is 43°. In other words a target at bearing 21 1/2° from the antenna axis gives one-tenth as much energy to the receiver as a target on the axis giving the same intensity of radiation.

The radiation pattern shown can not be used directly because there is no discrimination between right and left or up and down. The usual practice is to scan the field of view, to commutate or phase the received signal intensity with the scanning, and compare right with left and up with down or perform comparisons in some other coordinate system. The on-course indication becomes then an equality of two signals and a directional sense is provided. In Pelican, conical scanning is used, the antenna axis describing a cone of 11° half-vertex angle. A commutator provides the phasing. If there were a variety of targets of equal intensity, every one within a cone of half-vertex angle of approximately 22° would give signals of one-half maximum power or more. This represents one method of stating a figure for the field of view of the receiver, all targets being assumed to return signals of equal intensity.

In Pelican, the target is "illuminated" by a radar beam, and the directional characteristics of the transmitter antenna provide additional directional discrimination, which is not of interest in this discussion. In Bat, the missile carries the transmitter and the same antenna is used for transmission and reception. The field of view is accordingly smaller than for Pelican.

In other types of intelligence devices, much smaller fields of view are used, the width at 1/2 power being 10° or less. This is advantageous from the point of view of directional discrimination of targets. Limitation of the field of view is the first general method of securing target discrimination. However, a narrow field of view introduces tracking problems as will be discussed later.

The second method of securing target discrimination is by means of signal strength. This can not be entirely separated from the directional properties of the intelligence system, and permits little choice other than to home on the strongest signal. It is usually necessary to include some type of automatic gain control to obtain directional information at signal levels which may vary more than a billion fold as the missile approaches the target. The strongest signal within the field of view will govern the sensitivity of the receiver through the action of the automatic gain control.

Target discrimination may also be secured through the selective action of the intelligence device in responding to radiation within narrow wave length limits. This is best utilized when an intense beam of the desired radiation can be concentrated on the target and the missile made sensitive only to wave lengths within the narrow limits of the transmitted radiation.

In systems in which a pulsed illuminating beam is used, as exemplified particularly in the Pelican and Bat radar homing systems, another method of target discrimination may be used. This is range selection and synchronization. By making the intelligence system sensitive only for a short period at a predetermined time following the emission of an illuminating pulse, the control information can be restricted to that received from targets lying within certain range limits, say within a zone of ±250 feet of the actual target range. For a given range corresponds to a definite time of transit of the pulse from transmitter to target to receiver. Naturally such a range selecter requires an automatic method of tracking the target in range as the range decreases. The operation of the range selecter also requires the synchronization of the receiver and the transmitter. The synchronisation is effective in discriminating against reflected radiation originating from other transmitters operating on the same radio frequency but with different pulse rates.

The use of range selection is found to be essential in radar homing missiles launched from aircraft because of the so-called "altitude signal", i.e. energy returned from the earth directly below the aircraft. If a reasonable cone of vision is to be maintained, the directional selectivity of the antenna is inadequate to discriminate against the large reflecting area lying beneath the aircraft. A range selecter and range tracking device makes possible the elimination of this signal in the case of glider missiles, since the altitude is always less than the range to the target. Presently available radar homing devices can not be used under conditions where the target may appear at the same range as the altitude signal, for example in air to air missiles at ranges greater than the altitude. In the case of ground to air missiles using receiver only, the geometry is more favorable than for the glider, and the range of the target will not coincide with that of the altitude signal. For a send-receive missile, the missile will at some time be at the same altitude as the range to the target and hence may thereafter home on the altitude signal.

It is possible to devise more complex mechanisms to perform more difficult feats of discrimination, for example, to permit the launching of a radar homing missile at long range without advance selection of a target and to have the range selection device search for, choose, and lock on a target when the missile has proceeded a definite distance. The limit of performance is set only by the permissible complexity of the mechanism.

The suggestion has often been made that lower animals be used as intelligence devices, since their brain, like that of the human one, can perform difficult tasks of discrimination. This possibility is perhaps the only one of adequate complexity to deal with the pattern discrimination required to select, for example, a particular building within the complex optical radiation pattern presented by a city. Proponents of this method point out that near mechanical reliability may be realized in animals by establishing in them a conditioned reflex associated with the object selected for attention.

Relation between Field of View and Permissible Motion of Vehicle.

After a target has been selected by the operator before release of the missile or by the mechanism of the missile itself, the target must be tracked, i.e. held within the field of view of the missile during the remainder of its flight. The simplest method of tracking is to have the intelligence control the motion of the missile so that the target remains within the field. If this method of tracking is selected, restrictions are immediately placed on the permissible motions of the vehicle which must be considered by the aerodynamics specialist. These restrictions depend not only on the aerodynamics of the vehicle and the field of view of the intelligence device but also on target contrast, characteristics of tracking circuits, and behavior of the servo mechanism in the absence of homing signals.

When a missile is to be released blind, the aerodynamics specialist can compute trajectories for various release conditions and so provide estimates of the time at which a target in a specific location relative to the point of release will lie within a specified field of view. The relation between field of view, servo mechanism, and aerodynamical characteristics must be such that tracking will be preserved. The controls must be sufficiently effective to check any overshoot or the servo mechanism must have a memory to bring the vehicle and field of view of the intelligence device back on the target. For some types of vehicle, the aerodynamic design can be made such that the trajectory without any homing signal will include the desired target within the field of view. The larger the field of view, the easier the task.

Before release of the vehicle, the target must be brought into its field of view, and tracking in range established. During launching and thereafter the motion of the vehicle must be such as to maintain the target within the field of view of the intelligence device. Memory circuits within the intelligence head may allow the signal to fade out for short time intervals and resume tracking in range when the signal level is restored. If during the period of no signal return, the vehicle removes the target from the field of view, the intelligence device is unable to secure further information of target direction. The permissible motion will depend upon the intensity of the signal returned by the target. If the target signal is only a small amount of the background signal, the effective cone of vision is reduced (in present radar homing equipment) to about 70 percent of its maximum width. Thus a smaller change in attitude will be required to lose directional tracking than if the target signal is much larger than the background. Since the average signal level and its consistency in amplitude depends on target size, target orientation, and on meteorological conditions, it is difficult to give definite design rules. However, the smaller the field of view, the smaller the change in attitude required to reduce a low signal to the background level, and from these considerations a large field of view is desirable.

The behavior of the servo mechanism in the absence of signal or more exactly just following fading of the signal has a definite bearing on the relation between field of view and permissible motions of the vehicle. If the servo mechanism maintains the vehicle on the course it was flying, the field of view could be small without risk of losing the target outside of this field should the signal fade for a few seconds. If however the servo mechanism maintains the rate of turn and pitch which exist at the time of signal fading, the target would probably pass outside a small field of view before the signal returned. Either of these types of performance of a servo mechanism is somewhat idealized and not accurately obtainable in any actual mechanism. The maintenance of the same course is of advantage when the vehicle is nearly on the desired course and loss of signal is due to fading.

When the vehicle is initially coming on course, it may overshoot by a sufficient amount to lose track. If this occurs, a servo mechanism which maintains the course of the vehicle at the time the signal is lost will thereafter give no opportunity for again picking up the target signal. The amount of overshoot permissible will be larger, the larger the field of view and the greater the target contrast.

Some of the restrictions on the motion of the vehicle which are imposed by a narrow field of view can be removed by the use of an intelligence device fitted with automatic directional tracking. In this system the output of the intelligence device is used to drive servo motors to center the field of view on the target independent of the motion of the vehicle. The control of the vehicle itself is then derived from the relative position or rate of motion of the intelligence device with respect to the vehicle, or both. The minimum permissible field of view is then limited only by the precision and speed of response of the servomechanism. The extra degrees of freedom may give rise to more difficulty with stability of the two servomechanisms, one driving the intelligence, the other the vehicle. There has, as yet, been no field experience with a missile control of this type.

Background Signal.

All electronic intelligence devices have a certain internal noise level which can not be less than that produced by thermal agitation of electrons in the input circuit. The magnitude of this internal noise sets a lower limit to the signal which can be detected. However in actual practice there is a much higher background signal representing the signal return from areas other than that of the target which also lie within the field of view. Thus in a radar homing device the background signal is the reflected radar energy from the land, rough sea, or other obstructions that happen to be at the same range as the target. It may be very small or zero when the target is an airplane and the background is cloudless sky. In an optical homing device the background signal is the reflected or emitted optical energy from land, sea, or sky.

The important attribute of this unwanted background energy is its variability not only from place to place but at the same place, especially with weather. In the radar case against ship targets, the "sea return" depends very greatly on the height and shape of waves and on the orientation of the wave troughs with respect to the receiver. The performance of a radar homing missile against a specified target is affected very much by the condition of the sea surface, the permissible range at release being reduced as the sea becomes heavier because signal returned by the ship is lower and the amplitude of signal fluctuation is greater. Small targets may be lost in the sea return regardless of their range. Similarly, missiles using other parts of the electromagnetic spectrum encounter background signals which usually depend greatly on meteorological conditions.

The ratio of target to background signal may show large short-time variations during the flight of a single missile, and if the target contrast is not sufficient may produce difficulties in tracking at long range.

Fluctuation of Signal Intensity.

As a homing missile approaches its target, the signal intensity and usually the background signals also increase very greatly, making necessary an automatic gain control in the electronic equipment. The time constant of the gain control must be short to take care of the rapid change at the end of the flight but not so short as to obscure the variations of scanning frequency which give the directional information. The reliability of the information obtainable is dependent on the target contrast which is a function both of the background signal and the strength of the target signal itself. In addition to the slower variation of signal strength as the missile approaches the target, there may also be more rapid variations associated with the changing geometrical relationship between missile and target produced by the motion of the missile and the linear and angular motions of the target. Such fluctuations are always found in radar reflections.

The presence of fluctuating signal intensity and fluctuating background signal means that the missile and servo mechanism can not be designed on the assumption that information as to the angular bearing of the target is continuously available.

The effects of fluctuating signal intensity which must be guarded against are possible resonance effects in the servomechanism, synchronization with the frequency of scanning, and loss of tracking. It is not practical to lay down methods of design. However, in the radar case the M.I.T. field experiment group working on Pelican and Bat have found it advantageous to make photographic records of variations of signal strength of actual targets, to construct a special signal generator which emits variable signals controlled by a cam cut to the observed variation, and to test the effects of such a signal input on the intelligence device output.

III. <u>Maneuverability of Vehicle and other Aerodynamic Problems. The Six Degrees of Freedom</u>.

The trajectory of a missile is determined by the force of gravity and the reactions between the missile and the air through which it flies. The force of gravity acts vertically downward through the center of gravity of the missile. It is convenient to represent the resultant of the aerodynamic reactions by three mutually perpendicular force components acting through the center of gravity and three moments acting about the three axes along which the force components are taken. These forces and moments on a given missile are functions of the shape of the missile, the orientation relative to the direction of motion of its center of gravity, the speed, the axis and amount of angular rotation, and the density and other physical properties of the air.

The missile has six degrees of freedom, three linear and three angular. The interest of the user of a missile lies essentially in the three linear degrees of freedom, i.e. in the linear motion of the center of gravity of the missile. The angular motion of the missile is of interest only in so far as it modifies the three force components and thus the trajectory. A spinning or angular hunting motion is of no interest if the missile strikes the target, a result dependent only on the path of the center of gravity. It is generally true for the ordinary bomb, propelled aircraft, or glider that an absence of angular motion gives a more predictable and constant trajectory. But in some other missiles, for example, shells, a spinning motion is deliberately provided to provide a more stable and predictable trajectory.

The dynamics of a missile, even if restricted to airplane-like or bomb-like objects, is a very complex subject, and hardly appropriate for this report. Readers are referred to Vol V of W. F. Durand's Aerodynamic Theory for a discussion of airplane dynamics by B. Melvill Jones. Only elementary and general aspects will be discussed here.

The path of the missile can be controlled in a number of ways, but the most usual method is through changes in the aerodynamic reaction by means of changes in shape of the missile. The other methods may be advantageous in special cases. Thus by the use of rocket motors it is possible to apply reaction forces on the missile to change its path. This method is operative in the stratosphere where the air density is very small and in free space. Control requires the ejection of a part of the missile, a process differing only in degree from the burning of fuel to produce power to operate other types of control devices. Where other types of control are possible, it is usually more economical to use them.

A missile may be controlled by varying its mass distribution to modify the position of the center of gravity, thus changing the resultant moments of the air reaction, hence the orientation of the missile, which in turn modifies the force components. This was done in early airplanes by motion of the pilot but the method has been little used since that time.

It is possible to use power driven devices such as propellers or turbojets or to use thermal jets to produce forces to modify the path or to use such devices to apply moments to the missile to change its orientation and thus produce forces to modify the path. It is more common to use these devices as propulsion elements, and some control of the path, especially changes in the vertical plane, is accomplished by varying the propulsion force.

The most common method of control is through changes in shape of the missile which usually alter the moments of the air reaction and change the orientation of the missile in addition to modifying the force components directly. The use of a power driven propeller may be regarded as a special case of a periodic change in shape. All changes of shape for purposes of control involve the application of power, which may be derived from any of the usual types of power sources. It is obviously desirable that the power required for control be small. This has led to the conventional type of control used on aircraft in which the primary effect of the controls is to apply moments to the missile which in a time determined by the angular moments of inertia and the magnitude of the applied moments changes the orientation of the missile to the direction of motion of its center of gravity. As a result of the change in orientation the forces are changed and the path of the center of gravity is modified. We shall see later that there are advantages in the design of homing aeromissiles in selecting a change in shape which produces little or no change in moment but does produce directly changes in the force components. Such a method requires greater power for operating the controls than the conventional method.

Equilibrium and Trim.

An unpropelled missile can be in complete equilibrium only if the moments of the air reaction about three mutual perpendicular axes through its center of gravity are zero and if the resultant air force is equal to the weight of the missile and acts vertically upward. Such a state does not exclude the possibility of a spiral or spinning motion. In fact the tail-spin of an aircraft is a steady motion in which the above conditions are fulfilled. We shall however not consider such motions further, although there is no logical necessity of excluding spinning missiles. The guiding of such missiles would seem to introduce many technical complications.

The equilibrium of a propelled missile differs only in that the resultant aerodynamic force must equal the resultant of the weight and the propelling force.

Practically all missiles now used or under consideration have one or two planes of symmetry and a longitudinal axis which lies within 10° of the intended direction of motion. The exact location of the longitudinal axis is usually chosen to suit the convenience of the specific problems but always in a plane of symmetry. The other mutually perpendicular reference axes are called the lateral and normal axes, and if the missile has only one plane of symmetry, this plane contains the longitudinal and normal axes. In the practical construction of aircraft or missiles it is found impossible to make the device sufficiently accurately to insure that, when flown or released, the aerodynamic moments about the reference axes will be zero. It is always necessary to apply control moments of suitable magnitude about all three axes, or if it is desired to have the controls in a given neutral position with no force applied to the control levers, to provide adjustable trim tabs. These adjustments are easily made when a human pilot is on board, but other provision must be made when unmanned missiles are to be used.

For unbalanced moments about the lateral and normal axes, a stable missile compensates by angular rotation to new positions of equilibrium, since displacements about these axes produce restoring moments. The missile would then fly at a somewhat different angle of attack than planned and at an angle of yaw which would give rise to a lateral force producing a lateral drift of the missile. An unbalanced moment about the longitudinal axis, which lies approximately in the direction of motion, can not be compensated in this way, because rolling about the direction of motion produces no static restoring moment. The only methods known of compensating this unavoidable and undesired moment arising from lack of symmetry in the actual unmanned missiles involve automatic trimming by control surface displacements under the control of one or more gyroscopes. The most desirable method is to compensate the undesired moment directly by displacement of ailerons. In some cases, for example the German robot bomb V-1, the autopilot moves the rudder thus forcing the bomb to travel at an angle of yaw sufficient to produce a rolling moment due to yaw equal to the unbalanced rolling moment. This method of correction gives rise to a lateral drift which is one source of error contributing to the dispersion. One of the results of the early work on the Pelican and Bat developments was the demonstration that provision must be made in the autopilot for compensating for undesired aerodynamic moments arising from lack of symmetry, i.e. "trimming" the missile, and that a gyro or equivalent reference is essential. It is fairly well known that pendulums or aerodynamic surfaces whose position is controlled by aerodynamic reactions are ineffective for this purpose. In the Azon and Razon developments it has also been found desirable to introduce automatic trim devices to eliminate rolling motions, although not there required for stability reasons, since these missiles have two planes of symmetry. The elimination of the rolling motion in those missiles simplifies the control problem as will be discussed in the section on coupling between controls.

The state of complete equilibrium is rarely attained in the practical use of missiles. The linear motion of the center of gravity is an accelerated one, the mass times the acceleration being equal to and in the direction of the resultant force. The time required to reach equilibrium is often much greater than the time of flight of the missile. For example, a falling bomb is in complete equilibrium only when it reaches its terminal velocity, a process requiring fall from a great height and many tens of seconds. This long time constant arises from the limited rate at which energy is supplied from the gravitational field. In the case of an aircraft the slow phugoid oscillation arising from interchange of kinetic energy and potential energy has a period of approximately 0.22V seconds when V is the speed in ft/sec, i.e. 88 seconds for a missile traveling at 400 ft/sec. The design of servo mechanisms and intelligence devices can not be based on the assumption of equilibrium conditions.

Fortunately the time constants of the angular motions are much shorter, usually of the order of a fraction of a second or at the most a few seconds, increasing somewhat with the size of missile but decreasing with its speed.

Magnitude of Lateral Forces, Angular Rates, and Radii of Curvature.

Consider a symmetrical missile falling vertically with its longitudinal axis also vertical. The forces acting are the force of gravity and the air resistance. Because of symmetry there are no lateral forces. The downward acceleration will be equal to the difference between the acceleration of gravity and the ratio of the air resistance to the mass of the missile.

The acceleration will ultimately approach zero as the missile approaches its terminal velocity at which the air resistance equals the weight. At any point along its length the trajectory may be modified only by introducing a lateral force. This force imparts a lateral acceleration which causes the missile to travel in a path which is approximately circular for some time. To move the missile in a path of radius R requires a centripetal acceleration of V^2/R where V is the velocity of the missile. The rate of change of direction of the trajectory $\frac{d\theta}{dt}$ equals V/R.

The usual method of securing a lateral force is to change the orientation of the missile so that its axis makes an angle to the trajectory. The missile does not then travel in the direction of its axis. The change in its direction of motion is dependent on the magnitude of the lateral force produced by the change in orientation in relation to the mass of the missile. If the force is small or the mass is large, the trajectory will be modified very slowly, even though the axis of the missile is at a large angle to the direction of motion of the center of gravity. The missile behaves in the same manner as an automobile traveling on ice when the steering wheel is suddenly turned.

Experience from tests on bombs and airfoils shows that the lateral force produced at a given angle of the missile to its trajectory is approximately proportional to the square of the speed and to the projected lateral area. A reasonable value of the lateral force is about 10 lb/ft^2 on the fins or wings at a speed of 100 ft/sec and about 1/2 lb/ft^2 on a body of revolution at a speed of 100 ft/sec for angles of yaw of 15°, although with larger angles of yaw still higher values can be obtained. Extremely large angles of yaw give large drag forces which slow up the missile and thus reduce the lateral force. The lateral acceleration to be expected is therefore about $\frac{10A}{W}\left(\frac{V}{100}\right)^2 g$ where A is the area of the fins or wings in square feet, V the speed in ft/sec, W the weight of the missile in pounds, and g is the acceleration of gravity in ft/sec^2. In any actual design, the lateral acceleration should be determined from wind tunnel measurements on a model of the missile. A radius of curvature R requires an acceleration V^2/R. Hence

$$V^2/R = \frac{0.0322AV^2}{W}$$

or

$$R = 31\,\frac{W}{A}.$$

The rate of change of direction $\frac{d\theta}{dt} = \frac{AV}{31W}$.

For the standard 2000 lb bomb, A for the standard fins is about 4 square feet. By a suitable rudder, it may be expected that lateral forces of the above magnitude may be reached, in which case R = 15,500 feet and $\frac{d\theta}{dt} = \frac{V}{15500}$ rad/sec. = 0.0037 V degrees/sec. In the next 500 feet of fall after the rudder is applied, the bomb would move laterally about 7 1/2 feet.

It is to be noted that for this case of a vertical trajectory, the radius of curvature obtainable with a given fin area does not depend on the speed, since both the required and the available acceleration vary as the square of the speed. However the forces which the missile structure must withstand increase proportional to V^2/R.

When the trajectory makes an angle to the vertical, the force of gravity has a component at right angles to the trajectory. It is customary to measure the angle from the horizontal and to denote its value by θ. The gravity component is then $g \cos \theta$. The path then curves downward unless a sufficiently large lateral aerodynamic force overcomes the gravitational component. Calling the lateral aerodynamic force L, we have the following equation for the radius of curvature of the path

$$m\, V^2/R = mV \frac{d\theta}{dt} = mg \cos \theta - L$$

Consider first the case in which L is zero, i.e. a conventional bomb. We find $R = \frac{V^2}{g \cos \theta}$ and $\frac{d\theta}{dt} = \frac{g \cos \theta}{V}$. The maximum value of $\frac{d\theta}{dt}$ occurs when the axis of the bomb is horizontal and equals $\frac{g}{V}$ rad/sec. At a speed of 320 ft/sec, $\frac{d\theta}{dt}$ = 0.1 rad-sec = 5.7°/sec, and R = 3200 feet. As the speed increases, $\frac{d\theta}{dt}$ decreases and R increases.

Next assume that the trajectory of the missile is to be approximately a straight line to the target as is usually desired for a homing missile. In this case the average value of L must equal mg cos θ. The available control then depends on the changes in L which can be effected by the controls, and the control is the same as for a vertical trajectory. However, the requirement L = mg cos θ is the requirement for rectilinear flight and the speed required is dependent again on the area of fins or wings. The minimum speed for rectilinear flight, assuming the use of the maximum control, is given by

$$\left(\frac{V}{100}\right)^2 \frac{10Ag}{W} = g \cos \theta$$

or $$\frac{V}{100} = \sqrt{\frac{W}{A} \frac{\cos \theta}{10}}$$

For the 2000 lb bomb on a 45° trajectory, V is about 600 ft/sec.

At lower speeds than the minimum speed for rectilinear flight, the rate of change of direction is not as great and the radius of turn is larger. Values for any particular case can be estimated. The control obtainable depends greatly on the ratio of the speed to the rectilinear flying speed, and the rectilinear flying speed is determined mainly by the area of fins or wings.

Non-lifting Missiles.

From the aerodynamic point of view the properties of missiles in which the average value of the lateral force is zero are so different from those for which the average value is not zero that the two groups deserve separate treatment. Although an aircraft or glider could be trimmed to give zero lift, its general flight behavior would not be satisfactory, and the non-lifting missile usually takes the form of an elongated object with two planes of symmetry or more, and in some cases a body of revolution. The simplest example is an ordinary bomb or rocket stabilized by tail fins. When the axis is inclined, there is a restoring moment because the line of action of the resultant force passes behind the center of gravity. To keep the axis at an angle to provide a lateral force, this moment must be balanced by a smaller force in the opposite direction applied by a rudder at the tail or a spoiler at the nose. The action of the rudder or spoiler must be such that the missile still has sufficient static and dynamic stability about the new position of equilibrium.

Missiles of this type are best adapted to steep trajectories for the following reasons. Launching speeds are usually limited to a few hundred feet per second. Control against the deflecting action of gravity is effective only near the rectilinear flying speed. The minimum rectilinear flying speed is of the order of $100\sqrt{\frac{W}{A}\ \frac{\cos\theta}{10}}$. As the path approaches the vertical $\cos\theta$ decreases, the minimum rectilinear flying speed decreases, and the control improves.

A decrease in $\frac{W}{A}$ also decreases the rectilinear flying speed and hence improves the control at low speeds. This principle has been used in the Roc project in which additional surface has been provided to give larger lateral forces for a given angle to the trajectory.

entation) in a straight path which is inclined at an angle to the vertical. If propelled, the path may be inclined upward. Even if the controls are maintained in the neutral position, the missile will finally approach an equilibrium state of unaccelerated fall vertically downward at its terminal speed. For an ordinary bomb or other missile of high wing loading the time required corresponds to fall through a very great height.

The lifting missiles are intended to follow an approximately straight flight path, which for powered missiles may be horizontal or inclined upward. We shall consider first an unpowered missile, i.e. a glider.

The only forces acting on a glider in flight are the force of gravity and the resultant air force. It is customary to consider the resultant air force in terms of its components perpendicular and parallel to the direction of motion, the lift and the drag. In equilibrium gliding flight the resultant of lift L and drag D must balance the weight and hence must act in the vertical direction. The flight path is therefore inclined downwards at an angle θ such that $\tan \theta = D/L$. The resultant force R is usually expressed in terms of the dimensionless coefficient C_R defined by $R = C_R A \, 1/2 \rho V^2$ where A is the wing area, ρ the air density, and V the flight speed. Since in equilibrium the resultant equals the weight W the equilibrium speed of the glider along its flight path is determined by the relation

$$V = \sqrt{\frac{W}{A} \frac{2}{\rho C_R}}$$

For an airplane the value of C_R increases from a small value at 0° or some small negative angle of attack depending on the shape of the wing section nearly linearly up to angles of the order of 12 to 15°, reaches a maximum value of the order of 1.2 to 1.4 and then slowly decreases. The ratio of lift to drag reaches a rather sharp maximum value at some angle between 5 and 10° and then decreases rapidly. The slowest equilibrium gliding speed corresponds to the maximum value of C_R near the stalling angle and in this region of angle of attack the glide angle changes rather rapidly but with only small changes in equilibrium speed. As the angle is reduced the glide angle becomes flatter and the equilibrium speed increases. On passing the angle of maximum L/D the glide path again becomes steeper and the equilibrium speed increases still more. The steepest path is the vertical dive in which the equilibrium speed reaches its maximum value, the terminal speed at which the weight is balanced by the drag.

The preceding description applies solely to steady state conditions, i.e. those which occur after the lapse of a sufficient interval of time. It is important to observe that an increase in angle of attack at angles beyond that of maximum L/D at first flattens the angle of glide or even causes a temporary ascent and the steeper glide angle is obtained only after the lapse of sufficient time.

Let us suppose that while the airplane is gliding steadily at some angle of attack the angle is suddenly changed to a new value. The lift at angles below the angle of attack for maximum L/D is changed much more than the drag and the principal effect will be to produce an unbalanced force nearly perpendicular to the direction of motion which will curve the flight path. Ultimately of course the unbalanced drag force will modify the speed but this process requires some time. Suppose the lift coefficient corresponding to the steady straight flight at the instantaneous altitude and speed is C_{L_o} and the actual lift coefficient is C_L. The unbalanced force is then $(C_L - C_{L_o})\, A\, 1/2 \rho V^2$ and hence the lateral acceleration will be $\frac{(C_L - C_{L_o})\, A\, 1/2 \rho V^2}{W/g}$. The radius of curvature of the path will be $\frac{W}{(C_L - C_{L_o}) A g\, 1/2 \rho}$. This corresponds to the value 31 W/A given earlier if $C_L - C_{L_o}$ is taken as 0.64. For a given wing loading and air density the radius of curvature of the flight path depends on the difference between the actual value of the lift coefficient and the value which would be necessary in steady flight.

The minimum radius is obviously obtained with $C_{L_o} = 0$ and C_L equal to the maximum lift coefficient, i.e. with a non-lifting missile. The use of a lifting missile increases the minimum radius of curvature and hence gives lower manoeuverability.

For angles of attack beyond the angle of maximum L/D the effect of the drag may predominate and the speed may be reduced so rapidly that the lift force is not increased, although the lift coefficient is greater.

The foregoing discussion applies to control of the path of the missile in a vertical plane. The lifting missile usually has larger surfaces, i.e. wings for support and control in the vertical plane. For right-left steering the lifting missile is usually designed to use the airplane method, i.e. banking or rolling the missile to obtain a component of the force on the large wing surfaces in the desired direction. The turns which can be produced without banking are of very large radius. An airplane with dihedral angle will automatically bank if the rudder is turned and ailerons are not moved, and will turn to right or left if the airplane is rolled either by deflecting the ailerons or by turning the rudder.

Turns of a glider are descending spirals. If the inclination of the spiral flight path to the horizontal is θ, and the radius of the spiral is r, the radius of curvature R of the flight path is $r/\cos\theta$. Call the angle of bank ϕ and the lift L. The available force is then $L \sin\phi$ and hence

$$L \sin\phi = \frac{W}{g}\,\frac{V^2}{R}$$

V being the flight speed, W the weight of the aircraft, and g the acceleration of gravity. Setting $L = C_L\, 1/2\, \rho\, AV^2$ and solving for R

$$R = \frac{2}{\rho}\,\frac{W}{A}\,\frac{1}{C_L\, g \sin\phi}$$

The radius of turn can be decreased by increasing C_L as the glider is banked. This is the method commonly used in aircraft to make a tight turn. If C_L is zero, i.e. a non-lifting missile, banking does not give rise to a turn, R being infinite. If C_L is not zero, i.e. a lifting missile, banking gives rise to a turn even if C_L is not increased by action of the longitudinal control. This is one of the essential differences between the aerodynamic properties of lifting and non-lifting missiles. We shall discuss further the right-left steering of lifting missiles by banking in the section on coupling of controls.

It is quite possible to design a lifting missile which could turn without banking. A large vertical surface would be needed and various practical difficulties arise.

Non-lifting missiles will probably have less difficulty with compressibility effects at high speeds. The chief difficulty with the lifting missile arises from the fact that the missile is unsymmetrical about the plane of the wings and the orientation is maintained by reactions on wings and tail whose moments about the center of gravity are equal and opposite. Compressibility effects first appear on the wings, usually producing diving moments and large changes in attitude. The non-lifting missile on the other hand is symmetrical and both body and fin moments are zero except when control is applied. Thus the trim position of the non-lifting missile is not likely to change with speed in the absence of control. The control will undoubtedly be modified by compressibility effects.

At the present time aerodynamic data at supersonic speeds are quite limited, consisting mainly of information on the drag of projectiles obtained in firing tests. Adequate supersonic facilities are now being provided and within the next year or two a great deal of the necessary basic research should be completed.

Powered Missiles.

The control of powered non-lifting missiles requires little further discussion. When no control is applied, the propulsive force acts in the direction of motion. Variation of the propulsive force changes the speed but not the direction of motion. When control is applied, the orientation may change, in which case the propulsive force has a component at right angles to the direction of motion of the center of gravity, increasing the lateral force available for control. This effect is usually not large.

In the case of powered lifting missiles, variation of the propulsive force constitutes a second and independent method of control in the vertical plane. Under equilibrium conditions the speed is approximately independent of the value of the propulsive force, satisfying the equation

$$V = \frac{2W \cos \theta}{A \rho C_L}$$

For horizontal flight the propulsive thrust T is equal to the drag D. If T is greater than D the missile climbs at angle θ such that $T = D + W \sin \theta$. The angle θ may be negative corresponding to descent if T is less than D. For more complete discussion reference should be made to one of the many books on the subject of airplane performance.

Homing Missiles.

In homing missiles, the apparatus contained within the missile locates the target with reference to some axis fixed in the missile. The information so obtained is not adequate unless this reference axis is always in a known relation to the flight path of the center of gravity of the missile. The simplest solution is obviously to make the reference axis coincide with the flight path, i.e. to have the intelligence device "look" along the flight path. The installation is especially simple if the application of control does not change the orientation of the missile with respect to its flight path. The securing of this result is one of the aerodynamic problems peculiar to homing missiles. The solution used in the Pelican and Bat projects was to change the lift of the wing by trailing edge flaps and to so locate the center of gravity of the missile and a fixed tail structure that the downwash effects on the tail produced moments counterbalancing the moments produced by the flap deflection. The Roc project adopted the same solution.

In the flight of an unpowered homing missile in still air against a stationary target, the flight path is approximately rectilinear. The initial speed is usually much less than the terminal speed in the straight glide path and hence the speed increases. To maintain the rectilinear flight, the lift must be maintained constant and equal to $W \cos \theta$. As the speed increases the lift coefficient must be decreased by the action of the controls.

It appears desirable to operate unpowered homing missiles within the range of lift coefficients lying between zero and that for maximum lift-drag ratio. As is well known the equilibrium flight path first flattens and then steepens as the lift coefficient is increased. There is accordingly a reversal of control as regards the final effect when past the maximum lift-drag ratio. Missiles are not usually in equilibrium on their flight path, and computation shows that the first effect of control is always that of the change in lift. If however the lift-drag ratio at the maximum lift coefficient permitted by the controls corresponds to a slope of path steeper than the actual path slope to the target, the control proceeds to its limit and stays there. The intelligence calls for a higher lift. A higher lift coefficient is obtained as the control moves toward its limit but also a higher drag coefficient. The drag reduces the speed so that the actual lift does not increase very fast and the flight path corrects very slowly, if at all. It is probably desirable that powered homing missiles should also be operated in the region between zero and maximum lift-drag ratio.

If the homing missile is powered, various combinations of control are possible. For example, the propulsive thrust might be controlled by a speed-sensitive device to maintain a constant speed in the later stages of the flight.

Stability Problems.

In theory a homing missile might receive sufficient control information to give stable flight without inherent stability of the missile in the absence of control. In practice, a missile must have satisfactory stability to maintain its flight in periods of intentional or unintentional absence of control information.

It is not practical to review the many aspects of the stability of missiles. The disturbed motions of a stable airplane-like missile in the absence of homing control consist of various combinations of a rapid heavily damped longitudinal motion in which the angular pitching motion is most prominent; a slow oscillation, the previously mentioned phugoid oscillation, in which the missile rises and falls with the speed decreasing and increasing; a rapidly damped rolling and yawing oscillation; and a slow spiral motion.

Under certain conditions the missile may pass from steady rectilinear flight to a steady spin. In a true spin, as distinguished from the spiral motion referred to above, the wing surface is stalled, i.e. meets the air at a large angle. The spinning motion is however a steady motion with the following balance of moments and forces;

1. The stalled wing rotates of itself at such a speed that the rolling moment is zero. (A wing at normal angles offers resistance to rolling; a stalled wing is in unstable equilibrium at zero rate of roll).

2. The aerodynamic pitching moment tending to reduce the angle of attack is balanced by the centrifugal pitching moment tending to increase the angle of attack.
3. The aerodynamic yawing moment is balanced by the centrifugal yawing moment. The spinning characteristics are greatly affected by the angle of yaw at which this balance occurs.
4. The airplane descends at such a rate that the vertical component of the air force equals the weight.
5. The horizontal component of the air force gives the requisite centripetal acceleration of the center of gravity towards the spin axis.

A missile can be made difficult to spin by a suitable disposition of tail surfaces to give large aerodynamic pitching moments and a favorable anti-spin equilibrium angle of yaw.

When a homing device is applied to an airplane like missile, the disturbed motions take on a somewhat different character. For example, considering only the longitudinal stability, the slow phugoid oscillation disappears and is replaced by a damping of any disturbance of the velocity along the flight path, the period and damping of the fast angular pitching oscillation are controlled in part by the static longitudinal stability of the missile and in part by the power of the control surface and the lag of the servomechanism, and there arises a second angular pitching oscillation of longer period which may in some cases degenerate into two aperiodic motions. As the missile approaches the target, the frequencies and damping constants change somewhat and an additional mode appears.

In a similar manner the slow spiral motion associated with the lateral stability of the free flying missile disappears. There are short and long period combined rolling and yawing oscillations where damping may be positive, zero, or negative and the long period motion may degenerate into aperiodic motions.

Some aspects of the stability of homing missiles will be discussed briefly in the section on System Stability or Hunting Problems.

IV. Coupling Between Controls

Interdependence of Controls.

A missile in flight has six degrees of freedom but if it follows conventional aircraft design it has only three controls unless it is propelled in which case it has also a throttle or equivalent thrust control. The three controls are usually movable surfaces to control the moments about three mutually perpendicular axes and there is no direct control of the linear motions. Unfortunately the three controls do not give independent effects. Thus the rudder produces a small rolling moment as well as a yawing moment which turns the aircraft to right or left. When yawed, there results a much larger indirect rolling moment arising from the aerodynamic reactions. The ailerons produce yawing moments as well as rolling moments and the yawing moments may be "adverse" i.e. the resultant yaw produces a rolling moment in a direction opposite to the desired rolling moment or they may be favorable. In extreme cases the ailerons may turn the aircraft to right or left without rolling it or the rudder may roll the aircraft with little yaw. The effects may change sign for the same aircraft at different speeds i.e. at different angles of attack or at different lift coefficients. The interaction between the rolling or yawing motion and the pitching motion is fortunately very small.

In the case of non-lifting missiles, operation of the left-right and up-down controls simultaneously produces rolling moments and hence it has been found that a non-lifting missile must have ailerons if rolling motions are to be avoided.

Correspondence between Intelligence Coordinates and Controls.

The intelligence devices generally available for use in homing aeromissiles give information on the bearing of the target to the right or left and up or down from some reference axis, i.e. an essentially two-dimensional presentation. In the case of radar devices, range information can also be obtained if desired. The missile however has three controls if unpropelled and at least four if propelled. There is evidently a problem in making connection between the two-dimensional output of the intelligence device and the control system of the missile.

In the case of a non-lifting missile, the obvious method of connecting the controls is to connect the two channels of the intelligence device to two control surfaces producing moments about two mutually perpendicular lateral axes which are aligned with the intelligence device. While the two control surfaces may be designed to give independent action about the lateral axes, the application of control about both axes simultaneously will produce rolling moments about the longitudinal axis. If the aerodynamic design were such that the longitudinal axis remained in the direction of motion, the roll would not be objectionable unless the rate was so high that the lag in the control system introduced phase errors. However most missiles of the zero lift type change orientation as the controls are applied. The roll takes place about the axis of the missile and introduces incorrect error signals in the intelligence device. The better solution is to stabilize the missile in roll by means of ailerons controlled from a suitable gyro system. Thus in the simplest method, the roll control is governed by a gyro, and the other two by the channels of the intelligence device.

There have been many ingenious suggestions for connecting the controls of a non-lifting missile to permit continuous rotation of the missile. It does not seem profitable to discuss them here. For remotely controlled as distinguished from homing missiles such devices become computers, often of complex design, for transforming from fixed to rotating axes, and metering the degree of control to be given by each of the two control surfaces.

In the case of a lifting missile of the airplane type, turns are accomplished by banking the airplane and it is not therefore desired to prevent roll. It is necessary either to reduce the number of control surfaces from three to two or to couple two of the control surfaces together to be controlled from a single intelligence channel. If the missile is powered, the throttle or equivalent power plant control may be arranged to be controlled by airspeed, altitude, or some quantity associated with engine performance as for example mixture ratio, peak pressure, temperature, etc. Combinations may be used but it has not been customary to use the throttle for up-down control of the flight path.

The more common two-control airplanes use the elevator-rudder or the elevator-aileron combinations. Both methods have been used in the design of missiles. With proper design reasonably satisfactory turns can be made either with ailerons or with rudder. For homing missiles, aileron control is believed to give somewhat smaller errors, since out-of-trim rolling moments can be balanced without introducing yaw. In either case, the use of two controls alone makes possible independent connections of the two intelligence channels.

It has already been pointed out that a gyro is required for trimming the missile in roll. It is desirable that this gyro also limit the maximum angle of roll; otherwise the missile is likely to turn over on its back when a control signal is continued for some time.

In automatic pilots for aircraft, the rudder and aileron are often controlled together for turns. Such a coordination is possible for one or at most a narrow range of flight conditions. For missiles intended to operate over a considerable speed range, the rudder displacement for a given aileron displacement varies, both with airspeed and angle of attack, and it has not seemed necessary to attempt the design of the necessarily complicated control. By suitable aerodynamic design of a two-control missile, the side-slip during turns can be made reasonably small.

Effect of Roll on Error Signals.

The lifting missile of the airplane type banks during a turn. The reference axes of the intelligence device are fixed with reference to the missile and hence rotate with the missile. The axis of rotation of the missile does not usually coincide with the direction of motion of the center of gravity of the missile. Hence the intelligence device no longer measures the up-down and right-left errors with respect to axes fixed with respect to the vertical.

It has been previously pointed out that an airplane is made to turn in its tightest circle by banking and then pulling back on the stick to increase the lift coefficient to its maximum value. The coupling between the up-down and right-left controls produced by banking automatically gives an error signal to the elevator or elevons in a direction to increase the lift coefficient, and thus accelerates the turn. At the same time the increased lift bends the trajectory of the center of gravity upward. The exact behavior of the system depends on the characteristics of intelligence device, servo-mechanism, and missile.

The error angles are readily computed for idealized motions. If the missile rotated about the direction of motion of the center of gravity, the effects amount simply to rotation of the axes of reference of the error signals. If the error angles referred to the original axes are δ_e in elevation and δ_a in azimuth, and if referred to axes rotated through an angle ϕ δ_e' and δ_a', we have the relations

$$\delta_a' = \delta_a \cos \phi - \delta_e \sin \phi$$

$$\delta_e' = \delta_a \sin \phi + \delta_e \cos \phi$$

If however the rotation occurs about an axis making an angle δ_{e_0} with the direction of motion of the center of gravity

$$\delta_a' = \delta_a \cos \phi - (\delta_e - \delta_{e_0}) \sin \phi$$

$$\delta_e' = \delta_a \sin \phi + (\delta_e - \delta_{e_0}) \cos \phi$$

In the general case the dynamics of a rigid body with six degrees of freedom and the aerodynamic characteristics of the control must be considered. Thus the axis of torque varies with the displacement of the control surfaces. The body rotates not about the axis of torque, unless this axis is also a principal axis of inertia, but about an axis intermediate between the axis of torque and a principal axis. Thus the instantaneous axis of rotation travels in the body and an integration process is necessary. Because of these complications it is not very practical to correct for these effects by inserting a computing device between the intelligence and the controls. One of the important consequences of the foregoing effect is that when there is an elevation error angle and the true azimuth angle is zero, an azimuth error is passed on to the controls equal to $-(\delta_t - \delta_{t_0})\sin\phi$. To fix ideas suppose that the target is high so that $\delta_t - \delta_{t_0}$ is positive and that the airplane rolls as for a right turn (positive ϕ). A negative azimuth error is then given to the controls, which tends to oppose the right turn. If however $\delta_t - \delta_{t_0}$ were negative, i.e. true elevation error zero but axis of roll above flight path or target is low, the azimuth error given to the controls would assist the turn. In the first case there would be a damping effect on oscillations; in the second a destabilizing effect which would promote hunting. In the usual lifting missile the effect is on the average a destabilizing one.

<u>Effect of Angle of Attack or Yaw on Error Signals.</u>

In the non-lifting missile, the application of controls pitches or yaws the longitudinal axis so that the intelligence no longer looks along the flight path. Using the same notation as before, the effect is given by the relation

$$\delta_e' = \delta_e - \delta_c.$$

The same relation applies to the elevation control of a lifting missile. It is in theory possible to interconnect the controls with the intelligence device to move the reference axes of the intelligence device to compensate for this effect. If under-compensated there will be a stabilizing action against oscillations and if overcompensated a destabilizing action.

Methods of Reducing Coupling Between Controls.

The methods of reducing both the interdependence of the controls and the undesired effects of the angular motions on the error signals are still in the early development stages and much additional aerodynamic research is required. A homing missile can be made to work with all these effects present in some degree. It would seem fruitful however to attempt to make the normal flight path coincide with the longitudinal principal axis of inertia, for all control positions, and to do further research on reducing interactions between controls.

V. System Stability or Hunting Problems.

Illustration of Hunting.

Perhaps the most difficult of the special problems associated with the design of homing missiles is that of securing a satisfactory stability of the complete system, since the overall stability depends on the characteristics of missile, intelligence, and servomechanism and especially on their interrelations. To illustrate the problem there are shown in Figs. 2, 3, and 4 the observed motions of three homing missiles. In Fig. 2 it is seen that the missile rolls and yaws in increasing oscillations of about 10 seconds period until the missile strikes the ground. In extreme cases such a missile may turn completely over on its back, lose track of the target and descend in a steep spiral. In Fig. 3, the missile pitches in a steady oscillation of nearly constant amplitude and from 5 to 7 seconds period. Figure 4 shows a rolling and yawing oscillation which is damped as the flight progresses. The period of the main oscillation is approximately 10 seconds but there is superposed a rapid roll oscillation of about 4° amplitude and 1 1/4 second period.

The steady hunting motions illustrated in Figs. 3 and 4 are objectionable only in so far as they affect the path of the center of gravity of the missile. The unstable motion of Fig. 2 usually but not always results in complete failure of the missile and a wide miss.

The effect of a given hunting motion on the trajectory may be estimated as follows. The error angle is equal to the ratio of the velocity dy/dt of the missile transverse to the flight path to the velocity V of the missile, provided the missile looks along its flight path. If this angle oscillates sinusoidally with maximum amplitude θ_0, the motion may be represented by

$$\frac{dy}{dt} = V\theta_0 \cos \frac{2\pi t}{T}$$

where T is the period. The lateral displacement y is then given by the formula

$$y = y_0 + \frac{V\theta_0 T}{2\pi} \sin \frac{2\pi t}{T}$$

where y_0 is the mean value of y. The maximum excursion from the trajectory is then $\frac{V\theta_0 T}{2\pi}$. For $V = 400$ ft/sec, $\theta_0 = 3° = 1/20$ radian, $T = 2\pi$ seconds, the flight path oscillates approximately 20 feet. On the other hand if $T = 1$ second, the flight path oscillation is only 3 feet.

These simple computations underestimate the error since the actual missile does not look accurately along the flight path under dynamic conditions. This effect is however greatest for the short period oscillations where the errors are small.

<u>Factors Leading to Instability.</u>

The presence of error signals from a homing device would seem at first sight to remove most of the sources of instability and to supplement the aerodynamic damping forces which are present in any normal design. The element which introduces difficulty is the unavoidable lag between the presence of an error angle and the application of the corrective forces and moments to the missile. The lag may arise in the mechanical and electrical parts of the intelligence device, control system, or servomotor or may be introduced by the motion of the missile in response to the controls in a manner to introduce errors in the measurement of the error angle by the intelligence device.

The physical effect of the lag in producing hunting may be appreciated by considering the missile displaced from the correct flight path toward the target but being returned to it by the action of the controls. As the error angle approaches zero, the control surface deflection is approaching zero and should reach zero at the same instant as the flight path intersects the target. Or if the control is of the on-off type, reversal should occur at the instant the flight path intersects the target. If this could occur, the aerodynamic damping forces would absorb some of the kinetic energy present and the next crossings would occur with successively decreasing speed until the missile reached equilibrium on a steady trajectory. Because of the inevitable lag present, the control surface remains deflected in a direction to increase the error angle of the missile for a short time and thus feeds energy into the oscillatory motion. Equilibrium is reached at an amplitude of oscillation such that the amount of energy fed in is equal to that absorbed by the aerodynamic damping. It may happen that the equality of energy fed in and absorbed does not occur for any amplitude, in which case the oscillation builds up until the missile capsizes.

The amount of energy fed in is directly proportional to the time lag, and to the overall sensitivity i.e. to the magnitude of the correcting forces and moments produced by a given error angle. This overall sensitivity is dependent on such factors as the area of the control surfaces, the speed, altitude, and attitude of the missile, the deflection or rate of deflection of the control surface for a given electrical input to the servomechanism, and the sensitivity of the intelligence device.

The aerodynamic damping is a function of altitude and airspeed as well as of relative areas of various parts of the missile and of the amplitude of oscillation.

The time lag in the intelligence device is usually determined by the amount of integration or smoothing found desirable to give a reasonably steady output signal from the intelligence device. For the radar homing systems used so far, the time lag has been within the range 0.05 to 0.25 seconds.

The time lag of the [illegible] electromechanical servo system is of the order of 0.10 to 0.15 seconds, but considerably smaller values can be obtained by special design.

Either a time lag or a time advance may be introduced by the relations between the motions of the missile, the axis of the intelligence device, and the direction of motion of the center of gravity of the missile as discussed in the section on the effect of roll on the error signals. For the usual design of missile the axis of roll lies between the longitudinal axis of the missile and the direction of motion of its center of gravity and a time lag is introduced. Thus if a disturbance rolls the missile say for a right turn, a false azimuth error is indicated in a direction to aid the turn and amplify the effect of the disturbance. Or from the point of view of lag, if the missile is off course to the left and correcting to the right, the centering or reversal of control is delayed until the missile moves off course to the right a sufficient amount to bring the rotated reference axis on the target.

The use of automatic lead computers to reduce errors associated with wind and moving targets often leads to a difficult stability problem as discussed later in this report.

<u>Anti-hunt Devices</u>.

Since hunting is evidence of a phase lag between the error angle and the application of corrective forces and moments, the remedy is obviously the introduction by some means of a phase advance which is greater than the lag. As the missile comes on course, the control must be reduced or removed <u>before</u> the error is actually zero. The most common method of accomplishing this result is to incorporate a rate term in the control, i.e. to make the position or speed of the control dependent in part on the error and in part on the rate of change of the error.

A compensating or anti-hunt circuit can often be introduced in the amplifier or output circuit of the intelligence device itself. It usually takes the form of a suitable condenser-resistance network which may be considered either as a phase advancing device for sinusoidal signals or as introducing a rate component for arbitrary signal variation. Difficulties often arise with such a circuit if the input signals are "rough", i.e. fluctuating in magnitude for a given error angle as for example is the case for radar reflections from many types of targets. Because of the antihunt circuit, the higher frequency "roughness" is amplified much more than the slower variation as the missile comes on course. A further characteristic of this anti-hunt circuit is that good performance is obtained only with considerable attenuation of the input signal so that additional amplification is required.

The rate term may be introduced by a gyroscopic turn indicator which measures the angular rate at which the missile comes on course. This method has been found completely effective in obtaining system stability. It has the disadvantage that winds or moving targets require a steady rate of turn which introduces a control signal. The presence of this signal in effect limits the maximum rate of turn and thus increases the errors associated with winds and moving targets.

Time to Come on Course.

One of the characteristics of the control system of interest is the time required for a missile released with an initial error angle to come on its proper course. The time can be controlled to some extent by the overall sensitivity of the control system. It has already been pointed out that increasing the overall sensitivity increases the amount of energy fed into the system when a time lag is present. Even if the system is stable, the missile may come on course with a damped oscillatory motion and the damping is small if the sensitivity is too great. A return without oscillation can be secured with suitable values of the control parameters. Both experience and theory show that it is necessary to make a compromise between the stability characteristics and the time to come on course. In most cases it is necessary to make the time to come on course a matter of the order of 10 seconds or more.

Effect of Wind Gusts.

Wind gusts introduce disturbances which excite the natural modes of motion of the complete system. The effects persist for times which depend on the periods or time constants of the natural modes, and the disturbed motion at any instant depends on the history of disturbances over comparable past periods of time. Excluding the effects of changes in speed along the trajectory for which the time constant is very large but which in themselves do not cause the missile to miss the target, there usually exist modes for which the time constant is of the order of 10 seconds. Hence gusts in the latter part of the flight path introduce errors which can not be wholly corrected in the time available..

Experience has shown decreased dispersion in tests of homing missiles over water as compared with tests over land which is probably to be attributed to the decreased gustiness over water. In one instance over land, a severe disturbance was noted from the sharp boundary between a heated layer of air at the ground and a cold air mass aloft. While the designer of a missile can vary the frequencies of the natural modes of motion through small limits, it is practically impossible to change their order of magnitude. Fortunately the errors in the trajectory decrease as the rectilinear flight speed increases.

Methods of Analysis.

The problem of the design of stable systems has been approached by several methods of mathematical analysis and by experiments on mechanical, electromechanical, or electrical models. If sufficient information is available from tests of the component parts, the choice of control parameters to assure system stability can be made by any of these methods of analysis. The purely mathematical methods require more or less idealized representation of the performance of the component parts of the system and are most useful for systems whose performance is described by linear differential equations. The various methods using models have the advantage that some of the actual components can be incorporated as part of the model, so that non-linear control mechanisms and on-off or step controls whose mathematical analysis is often difficult can readily be investigated.

The control system of a homing missile constitutes a closed-cycle control system or servomechanism. The chief difference from ordinary servomechanisms is that the parameters of the system are not constant but vary through considerable limits during the flight of a single missile. More specifically the parameters associated with aerodynamic forces and moments are functions of the air speed, air density, and attitude of the missile and its control surfaces. If these variations are taken into account, the usual methods of analysis of servomechanisms may be applied. These involve a study of the response of the system to certain standardized conditions, the two most useful conditions translated in terms of a homing missile being a sinusoidal displacement of the target at various frequencies or a sudden permanent displacement of the target. If the system is a linear one, its performance is completely known when the performance under either of the above conditions is known.

Readers who are interested in the details of the mathematical procedures are referred to the restricted paper by A. C. Hall of the Servomechanisms Laboratory, Massachusetts Institute of Technology, entitled "The Analysis and Synthesis of Linear Servomechanisms", Technology Press, 1943. This paper describes the application of the sinusoidal analysis to the design of linear servomechanisms with continuous control. An example of analysis using suddenly applied disturbances may be found in Restricted Technical Note No. 809 of the National Advisory Committee for Aeronautics by F. H. Imlay entitled "The Theoretical Lateral Motions of an Automatically Controlled Airplane Subjected to a Yawing Moment Disturbance". Robert T. Jones of the same laboratory has developed this method of study of the control of certain types of missiles.

The preceding methods of analysis lend themselves to a complete study of the performance of the system, including not only the question of whether the system is stable or not but all questions of the magnitude of the errors. For determination of stability, one may apply the method of small oscillations commonly used in airplane stability problems. This method as applied to an airplane without automatic pilot is described in great detail by B. Melvill Jones, in Division N, Vol 5 of Durand's Aerodynamic Theory under the title "Dynamics of the Aeroplane". The method in effect determines the period and damping of the natural modes of oscillation and the damping of the natural aperiodic motions. If the damping constant turns out to be negative, the corresponding mode is unstable.

The mathematical methods are in practice limited to systems described by linear differential equations and find some difficulty in the treatment of actual servo systems with friction, "dead" regions, and time lag. For this reason models have been found useful. These range in complexity from systems representing a single degree of freedom of the missile to more complete flight tables which include the three angular components of the motion of the "phantasmagoria" which simulate one or two components of the linear motion. As an example, methods of automatic roll stabilization may be investigated with the actual gyro and servo elements by using a mechanical system (damped pendulum or damped rotor) to simulate the inertia and damping of the missile about its roll axis. A system of this type as well as a more complicated electro-mechanical model of the longitudinal motion were used in the study of the Pelican and Bat control systems. In addition, Dr. A. C. Hall of the Servomechanisms Laboratory, Massachusetts Institute of Technology, developed a flight table in which the pitching, yawing, and rolling motions of the missile with proper cross coupling were fully simulated and on which an overall test could be made of the complete control system including intelligence unit, gyroscopes, and servomechanism. This flight table was extremely useful in determining the best adjustments of the several parameters of the control system.

Experience with these model methods and comparison with actual flights of homing missiles show that the models reproduce the angular motions encountered in flight extremely well. Designers of homing missiles will find that the use of this method of studying system stability and overall performance will save much time in the development of a stable system and in adjusting it for best performance.

VI. Problems arising from Wind and Moving Targets.

Pursuit Curves

If the target is moving or there is a natural wind, a flying missile which always heads toward the target follows a path known as a pursuit curve or homing curve depending on whether the motion is referred to the air or to the ground. The pursuit curve has the characteristic that the radius of curvature becomes very small as the target is approached and becomes infinitely small for the idealized case of a point target. The maneuverability of the missile is limited and hence the missile will not hit a moving point target or a stationary target in a cross wind. The magnitude of the miss depends on the speeds of the missile, wind, and target and their relative directions, on the range, and on the maneuverability of the missile as expressed by its minimum radius of curvature.

This problem in idealized form has been studied by the Statistical Research Group, Division of War Research, Columbia University, under the direction of the Applied Mathematics Panel. For convenience the effect of wind and target motion are combined to give an apparent target motion of speed v. (See Figure 5). The azimuth ϕ_0 of the launching position referred to the direction of apparent motion of the target, the launching radius r_0, the ratio n of the apparent target speed v, to the speed of the missile V, and the minimum turning radius ρ_m of the missile are the quantities determining the miss for the case of point target. Constant velocities of target and missile, direction of launching toward the target, and missile continuing along an osculating circle when the minimum radius of curvature is reached are assumed.

The computed miss M is shown in Figure 6 in the form of a plot of M/ρ_m vs ϕ_0 for n = 0.1, 0.2, 0.3 and 0.4 and $r_0 = 6\rho_m$.

$10\rho_m$, and $14\rho_m$. The essential points to notice are (1) that the maximum miss varies but little with launching radius; (2) that the maximum miss is approximately equal to $1/2\rho_m n^2$ and (3) that the miss becomes very small for ϕ_o between 140° and 180°. The only factors within the control of the designer of the missile are the speed of the missile and its minimum turning radius. The speed ratio should be as small as possible and the minimum turning radius as small as possible to minimize pursuit curve errors. The user of the missile may reduce the error by making his attack on a course such that ϕ_o is approximately 145°. In the case of ship targets this course gives a good compromise between the magnitude of the error and the projected length of the ship which determines the permissible error which still yields a hit.

Computers.

The essential feature of the pursuit curve is that the missile moves along a path which makes an angle $\tan^{-1} \frac{n \sin \phi_o}{1+n \cos \phi_o}$ with the line joining the missile and target. If the axis of the intelligence device were rotated with respect to the longitudinal axis of the missile by this amount in the opposite direction to the apparent target motion, the path with respect to axes fixed in the target would be straight and the missile would travel on a "collision" course. If the user of the missile has the necessary data to compute n and ϕ_o, and the means of offsetting the axis of the intelligence device by the computed amount and if the variations of n and ϕ may be neglected, the pursuit curve error may be eliminated.

It is in theory possible to design a computer which automatically offsets the axis of the intelligence device by the required amount. The simplest scheme in theory is to equip the intelligence device with a separate servo system which always keeps the axis of the device centered on the target. If the missile flies a collision course the bearing of the target will remain constant and hence the angular velocity of the intelligence device will be zero. If the controls are so arranged that the missile is guided to make the angular velocity zero, the course followed should be the desired collision course. In practice a high precision is required since the rate of change of bearing for a small offset is very small at long ranges. Furthermore the missile is subject to numerous disturbances of attitude and it appears that an accurate linear integrating angular velocity meter would be required. It is not known whether the scheme would work or not; it has not yet been tried on a missile.

Another possibility which amounts to integrating the angular velocity is to determine the angular displacement of the axis of the missile after a certain time or distance by means of a free gyro which maintains a fixed direction in space. A knowledge of the angular displacement and the range at the beginning and end of the displacement permits a computation of the desired offset. This computation also assumes a constant value of n throughout the flight.

Still another possibility is a continuously integrating angular velocity meter which continuously sets in the appropriate offset.

Study shows that the roughest sort of correction reduces the pursuit curve errors even though a true collision course is not obtained. Hence it is worthwhile to experiment with the simplest conceivable computers, for example those dependent merely on the relative frequency and length of error signals in the two opposite directions.

The chief bar to the general use of computers lies in their effect on system stability and in fact the automatic devices described can not be used because they produce excessive hunting. The computer essentially puts in a lead on a moving target. If the missile as a result of a disturbance rotates to increase the lead, the angular velocity developed acts to increase the lead still further and thus builds up an oscillation of increasing amplitude. The computer can not distinguish between angular velocity arising from disturbances and angular velocity caused by the missile following a pursuit curve. An automatic lead computer is an inherently destabilizing device.

The two methods of solving the stability problem are (1) to feed in only a fraction of the correct lead angle or (2) to introduce a time constant in the integration process which is long compared to the natural periods of oscillation of the homing missile. In the first case the permissible fraction is dependent on the stability reserve of the complete system of intelligence device, servocontrol, and missile. In the second case the time constant must be of the order of at least 20 or 30 seconds for missiles exhibiting natural periods of the order of 1 to 8 seconds.

The computer problem is susceptible of mathematical analysis for any specific missile and control system, and is not too difficult if all elements of the system are linear. Actual experience with computers on a homing missile is not yet available.

VII. Bore Sight Errors.

If the axis of the intelligence device does not coincide with the direction of motion of the center of gravity of the missile, there arises an error which may be termed the bore sight error. Let us confine our attention to the case where the intelligence device is not equipped with a computer or other means of introducing an offset angle, i.e. the axis is fixed with respect to the missile. The missile then flies at a fixed angle to the line joining missile and target and hence follows a logarithmic spiral whose equation in polar coordinates is $\rho = \rho_0 e^{-\frac{\theta - \theta_0}{\tan \epsilon}}$ where ρ_0 and θ_0 are the coordinates of the release point and ϵ is the bore sight error. As the missile approaches the target the radius of curvature decreases and finally becomes equal to and would be less than that which the missile can follow. The radius of curvature of the path is equal to $\rho/\tan\epsilon$ or if ϵ

is small to ρ/ϵ. Thus if R_c is the minimum radius of turn of the missile, this value is reached at a value of ρ equal to $R_c \epsilon$. This value of the range to the target at which the radius of turn becomes R_c will be designated R_m. If it is assumed that the missile continues to travel in a circular path of radius R_c, the miss is readily computed to be $1/2 R_c \epsilon^2$. For if the path were straight the miss would be $R_m \epsilon = R_c \epsilon^2$ and the circular path produces a correction $1/2 R_c \epsilon^2$ leaving a residual miss of $1/2 R_c \epsilon^2$. For $R_c = 10{,}000$ feet, $\epsilon = 1°$, $R_m = 167$ feet and the miss is only 1.4 feet. The miss increases as the square of the bore sight error.

Bore sight errors arise from many sources in addition to the obvious one of inaccuracy of construction of the vehicle and of the mounting brackets of the intelligence device. Both mechanical and electrical imperfections of the intelligence device may give rise to bore sight error. Thus in a radar homing device the electrical axis of the antenna system may not coincide with the geometrical axis because of unsymmetrical distribution of dielectric or conducting material near the antenna. The output circuits may be unbalanced such that the output is not zero when the target is on the axis of the antenna. The servo control system may contain elements which are not balanced when the input signal is zero; for example, the pick-off of a rate gyroscope may be displaced from the correct zero position. Errors from these and similar causes may be controlled by careful inspection tests of the individual components and an overall check of an installed system can readily be made. The reference axis of the vehicle can be oriented with respect to the target until the indicated output is zero in which case the angular displacement of the target from the reference axis gives the bore sight error.

The most difficult problem is to determine that reference axis of the vehicle which lies in the direction of motion of the center of gravity of the vehicle in free flight. In fact a vehicle designed like a conventional airplane travels at different attitudes for different positions of the control surfaces. Particularly in the vertical plane the angle of attack varies with the elevator setting. Thus there is a variable bore sight error which might amount to as much as 10° or more.

A similar error may occur in the horizontal plane when rudder and elevator control are used. For in this case the rolling moment due to any lack of symmetry of the missile about its longitudinal axis must be balanced by application of rudder to yaw the missile which then flies at an angle to its longitudinal axis.

Two methods have been used to reduce bore sight errors from aerodynamic causes. The first and most satisfactory is to design the missile so that the change in attitude with application of control is as small as possible. This requires aileron-elevator control rather than rudder-elevator control, so far as the horizontal motions are concerned. In the vertical plane the aerodynamic design should be such that the lateral forces are modified without change in attitude. One method is to use flaps on the trailing edges of the wings to change the lift and to balance the pitching moments so produced by pitching moments from the tail arising from changes in the downwash angle. This can be done by proper choice of tail size and location and of center of gravity location of the missile.

The second method which has been used to reduce boresight errors from angle of attack changes is to mount the intelligence device on trunnions and couple it to the controls in such a manner that the reference axis of the intelligence device is rotated to compensate for changes in angle of attack. This compensation can be made perfectly for steady flight conditions but there are residual errors under dynamic conditions. In addition a stability problem arises as in the case of computers and usually only a partial compensation can be made if hunting troubles are to be avoided. In some instances such a coupling of intelligence device to controls has been used as a means of damping as previously discussed. In such a case the bore sight error is not completely eliminated.

A final source of bore sight error is the inevitable inaccuracy of construction of the vehicle itself as regards its external shape. The axis of zero moment i.e. the direction of travel will vary somewhat from one missile to another of the same intended shape. Such errors can be measured in a suitable wind tunnel but only with great expenditure of time and the requirements for uniformity of flow in the wind tunnel are difficult to meet. In the case of Pelican and Bat missiles, errors of about 0.8° were found from this source.

With sufficient care and neglecting actual equipment failures it should be possible to keep the bore sight error to the order of magnitude of 1°.

VIII. Effects of Sensitivity, Lag and Minimum Range of Intelligence Device on Pursuit Curve and Bore Sight Errors

The preceding discussion of pursuit curves and bore sight errors has assumed certain ideal characteristics of the intelligence device. The facts that the intelligence device must look away from the target to produce error signals and that it has a certain lag have been neglected.

Let us consider first an intelligence device without lag which gives an output signal proportional to the error angle up to a certain angle σ_m and which gives a constant output for angles greater than σ_m. The maximum output signal gives the minimum radius of curvature of the path ρ_m. For any error angle σ smaller than σ_m it will be assumed that the radius of curvature ρ is given by $\rho / \rho_m = \frac{\sigma_m}{\sigma}$; i.e. the radius of curvature is infinite for $\sigma = 0$. If then the discussion of the pursuit curve is reexamined, it is seen that the axis of the intelligence device must point away from the target by an angle σ when the radius of curvature is ρ.

An exact solution of this problem is difficult and has not been carried out. We have already seen that the pursuit curve error is entirely analogous to a bore sight error of magnitude $\tan^{-1}\frac{n \sin\phi_m}{1+n\cos\phi_m}$ where n is the ratio of the apparent target speed to the speed of the missile and ϕ_m is the azimuth of the missile referred to the direction of apparent motion of the target. This equivalent bore sight error varies during the flight of the missile along a pursuit curve because ϕ_m varies and the complication of pursuit curve calculations is the determination of ϕ_m as a function of the initial azimuth, initial range, and n. It will be instructive however to consider the simpler problems of the effect of the value of σ_m on the miss arising from a constant bore sight error ϵ.

This problem has been worked out by Dr. Skramstad (unpublished) for the case where ϵ is small, the lateral displacement y of the missile from a line joining the release point to the final target position is small compared to the initial range $x_0 - x$, and the slope of the trajectory $\frac{dy}{dx}$ to the same line is small compared to 1. The differential equation is found to be

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{\sigma}{\rho_m \sigma_m} = \frac{1}{\rho_m \sigma_m}\left[\left(\frac{y}{x-x_0}\right) - \frac{dy}{dx} + \epsilon\right]$$

The solution of this differential equation was found to be

$$y = \epsilon(x_0-x)\left[\log\frac{x_0}{x_0-x}\ \frac{H(x_0/\rho_m\sigma_m)}{x_0/\rho_m\sigma_m}\right] + \epsilon$$

$$\frac{dy}{dx} = \epsilon - \epsilon\log\left[\frac{x_0}{x_0-x} - \frac{H(x_0/\rho_m\sigma_m)}{x_0/\rho_m\sigma_m}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-\epsilon}{x_0-x}$$

where $$H(x) = 1 + \frac{1}{x} + \frac{2}{x^2} + \frac{6}{x^3} + \cdots\cdots$$

For practical values of x_0, ρ_m, and σ_m the second term in the brackets is entirely negligible. If at a range $x_0 - x = R_m$ the missile is assumed to follow a circle of radius ρ_m the miss M is given by

$$M = y + R_m \frac{dy}{dx} - 1/2 \frac{R_m^2}{\rho_m}$$

Whence

$$M = \epsilon (R_m + \rho_m \sigma_m) - 1/2 \frac{R_m^2}{\rho_m}$$

The range R_m at which $\sigma = \sigma_m$ or $\rho = \rho_m$ is equal to $\rho_m \epsilon$

since $$\frac{1}{\rho} = \frac{d^2y}{dx^2} = - \frac{\epsilon}{x_0 - x} = \frac{-\epsilon}{R}$$

Hence the miss is given by

$$M = \rho_m \sigma_m \epsilon + 1/2 \rho_m \epsilon^2$$

If we assume $\sigma_m = \frac{1}{15}$ we obtain the following values:

ϵ	$\frac{M}{\rho_m}$
.05	.0046
.10	.0117
.15	.0213
.20	.0333
.25	.0480

The effect of the value of σ_m is zero if $\epsilon = 0$. Thus if the missile travels a collision course and the bore sight error is zero, there is no error from limited sensitivity of the intelligence device. When ϵ is not zero, the effect of σ_m is to give an error term proportional to ϵ whereas the miss due to bore sight error alone is proportional to ϵ^2. Hence the effect of σ_m predominates at small values of ϵ .

It is possible to use the formulae previously given to compute the pursuit curve errors by means of an equivalent bore-sight error. For the azimuth angle ϕ_0 of the missile referred to the direction of the apperent of the target changes from ϕ_0 to $\phi = \phi_0 - \frac{dy}{dx}$. Introducing the value of $\frac{dy}{dx}$ in the expression for equivalent bore sight error, there is obtained

$$\epsilon = \frac{\sin\left(\phi_0 - \epsilon + \epsilon \log x_0/R_m + \epsilon \log 1/\epsilon\right)}{n + \cos\left(\phi_0 - \epsilon + \epsilon \log \frac{x_0}{R_m} + \epsilon \log 1/\epsilon\right)}$$

This transcendental equation can be solved for ϵ as a function of ϕ_0 , n and x_0/R_m and with the expression for M in terms of R_m, σ_m , and ϵ the values of $\frac{M}{R_m}$ as a function of σ_m , ϕ_0 , n, and x_0/R_m can be found. The accuracy is not very good when $|\phi - \phi_0|$ exceeds about 0.3.

Since large pursuit curve errors correspond to large values of ϵ , the term in σ_m is the smaller but for $\sigma_m = \frac{1}{15}$ corresponds to increasing the miss by 50 to 100 percent.

The effect of a time lag τ in the intelligence device may be estimated in a qualitative way by considering this effect also as an equivalent bore sight error. The error signal actually present at a range R is that appropriate to a range R + Vτ where, V is the speed of the missile. This may be regarded as due to an error angle at range R differing from that actually present by an equivalent additional bore sight error equal to the change in the error angle between $R + V\tau$ and R. This is readily seen to be

$$\rho_m \sigma_m \epsilon \left(\frac{1}{R} - \frac{1}{R + V\tau}\right) = \frac{\rho_m \sigma_m \epsilon V \tau}{R(R + V\tau)}$$

The total equivalent bore sight error is then $\epsilon\left\{1 + \frac{\rho_m \sigma_m V \tau}{R(R + V\tau)}\right\}$.

The range at which $\rho = \rho_m$ is then given by

$$R_m' = \rho_m \epsilon \left\{1 + \frac{\rho_m \sigma_m V \tau}{R_m'(R_m' + V\tau)}\right\}$$

This can be solved for R_m' and introduced in the equation previously given to find the corrected miss. For most cases in practice the total equivalent bore sight error may be approximated by

$$\epsilon\left(1 + \frac{\sigma_m V \tau}{\rho_m \epsilon^2}\right)$$

The magnitude of the additional error is not large.

If the intelligence device overloads as the target is approached, the directional information may disappear completely or the error signals may even be reversed in sign. Furthermore certain types, such as radar, have a minimum range within which no error signals are given. If the minimum range is R_m, the missile in this case continues in a straight line and the miss is given by

$$M = y + R_m \frac{dY}{dX} - \epsilon (R_m + \rho_m \sigma_m)$$

If for example R_m = 2000 feet as was found in one design of intelligence device, R_m = 10,000 feet, $\epsilon_m = \frac{1}{15}$ and $\epsilon = \frac{1}{30}$, the miss is 89 feet. If R_m is reduced to 200 feet the miss is 29 feet of which 7 feet is associated with the minimum range and 22 feet with the radar sensitivity ϵ_m.

IX. Strength Problems of Homing Missiles.

The structural design of a missile is in most respects entirely analagous to that of an aircraft. It is desired here only to call attention to two design conditions peculiar to missiles.

For rocket or jet propelled missiles it is often inefficient to design the power plant of sufficient size to include the take-off condition. It is better to use assisted take-off by means of a catapult or by special launching rockets. In German experience with anti-aircraft and long range missiles, accelerations from 2 to 16g have been used in launching. The missile structure must be designed to withstand the inertia loads produced, and so must all of the parts of the intelligence device and control system.

The second design condition peculiar to missiles is that of full control application at maximum speed. Because of the bore sight error which may be present, the intelligence device will call for the maximum rate of correction. The missile must be able to withstand the loads so produced or some automatic device must be used to limit the acceleration which may be imposed.

~~C O N F I D E N T I A L~~

X. Launching Problems of Homing Missiles.

Some discussion has been given of launching problems in the discussion of the relation between the field of view and the permissible motions of the missile. The target must come within the field of view and remain there. A sighting device or computer may be required for determination of the proper release time.

A special problem encountered in the release of missiles from aircraft is that of avoiding maneuvers which cause contact between missile and aircraft after release. It appears essential to keep the homing device disconnected from the controls until the missile is at a safe distance from the aircraft. The choice of air speed and attitude at release can be made after the study of computed trajectories confirmed by experimental tests. In general the air speed must be lower than the horizontal flight speed of the missile, the amount by which it should be lower depending somewhat on the drag-weight ratio of the missile. In some cases interference effects between the missile and the aircraft may cause an unfavorable trajectory. However no such effects have been encountered for missiles of high wing loading.

XI. Conclusion.

A survey has been given of some aspects of the design of homing missiles for flight through air, primarily to place on record that part of the experience in the study of the radar homing glide bombs, Pelican and Bat, which is likely to be of value in the future development of homing missiles. None of the topics have been treated in detail in this paper, but it is believed that the discussion is sufficient to indicate the nature of the problems and possible methods of their solution.

Fig. 1 Directional sensitivity of antenna of Pelican radar receiver.

Fig. 2 Unstable oscillation of homing missile.

Fig. 3 Undamped oscillation of homing missile.

Fig. 4 Damped oscillation of homing missile.

Fig. 5 Diagram of launching variables determining pursuit curve errors.

- r_o distance to target at launching
- ϕ_o azimuth of launching position referred to direction of apparent motion of target.
- v apparent speed of target, resultant of wind effect and target motion.

Fig. 6 Pursuit curve errors due to effects of wind, target motion, and limited maneuverability of missile.

- r_o distance to target at launching.
- ϕ_o azimuth of launching position referred to direction of apparent motion of target.
- v apparent speed of target, resultant of wind effect and target motion.
- V speed of missile.
- n speed ratio v/V.
- ρ_m minimum radius of curvature which missile can be made to fly.
- M miss referred to point target.

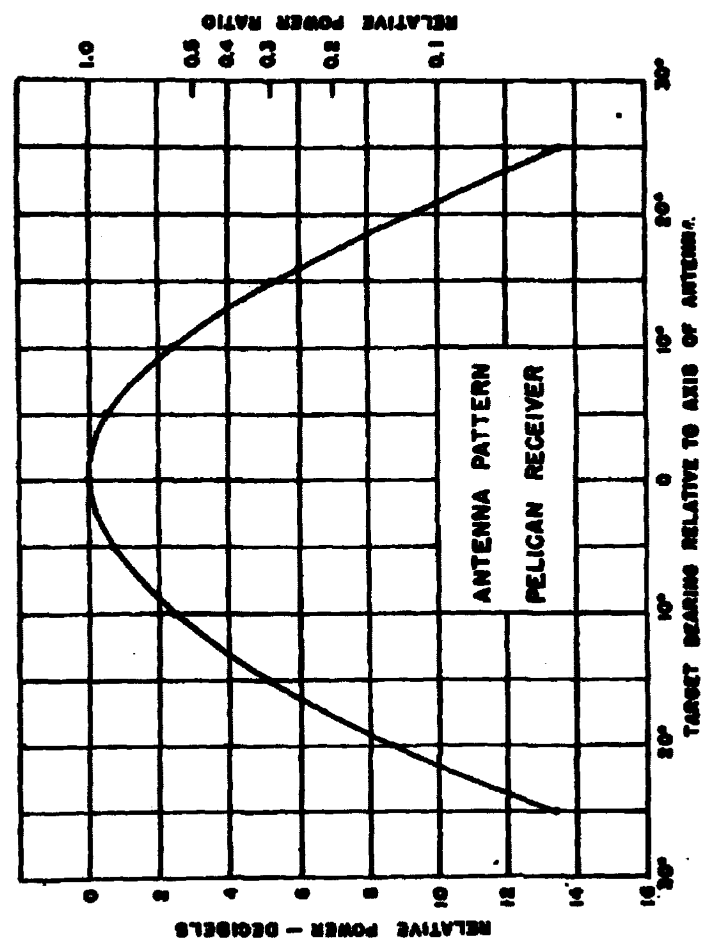

RELATIVE POWER RATIO
1.0
0.5
0.4
0.3
0.2
0.1
RELATIVE POWER — DECIBELS
0
2
4
6
8
10
12
14
16
TARGET BEARING RELATIVE TO AXIS OF ANTENNA
30°
20°
10°
0
10°
20°
30°
ANTENNA PATTERN
PELICAN RECEIVER

Fig. 1

-79-

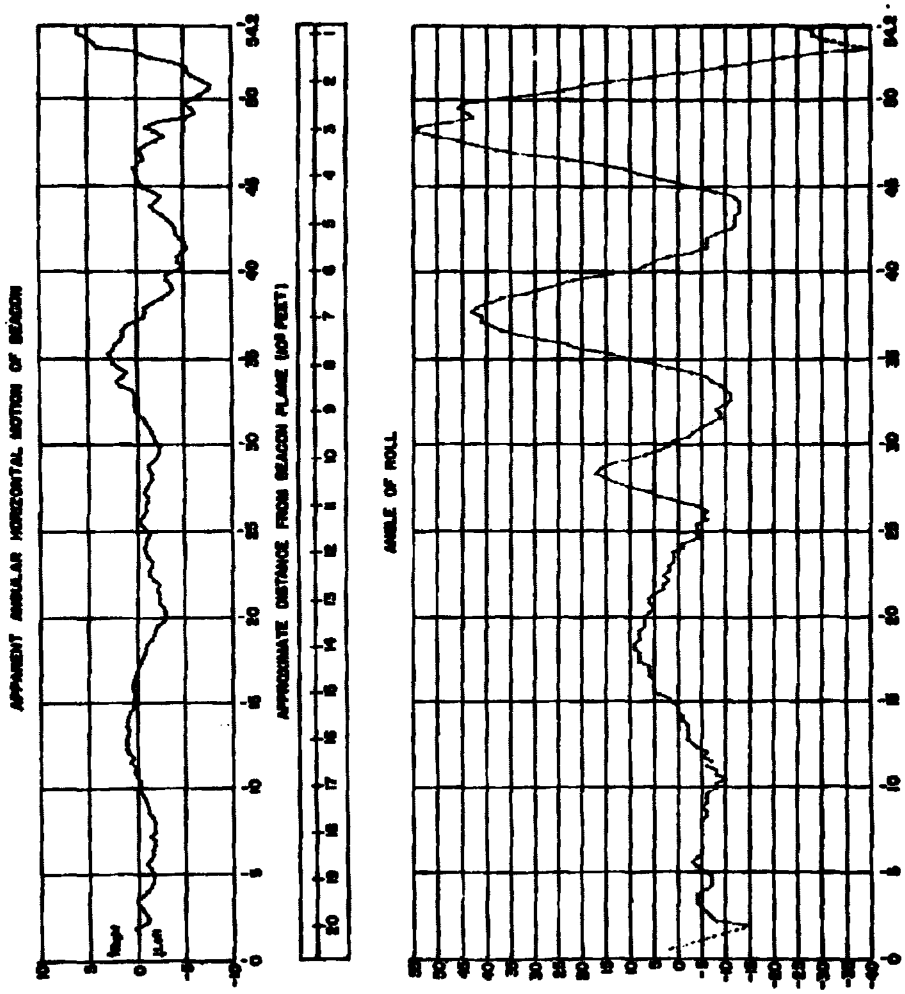
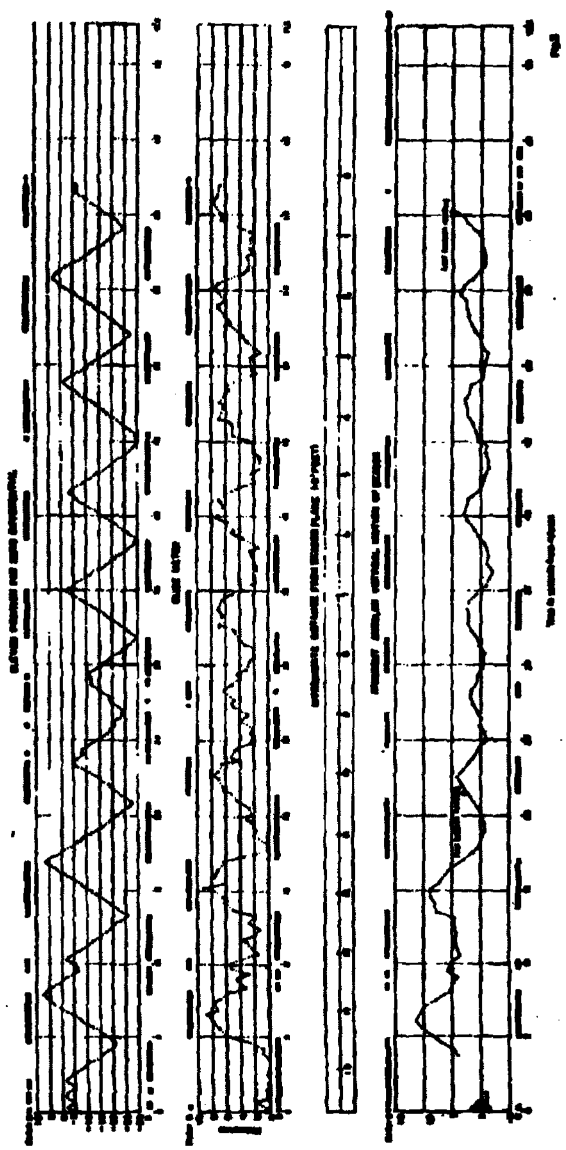
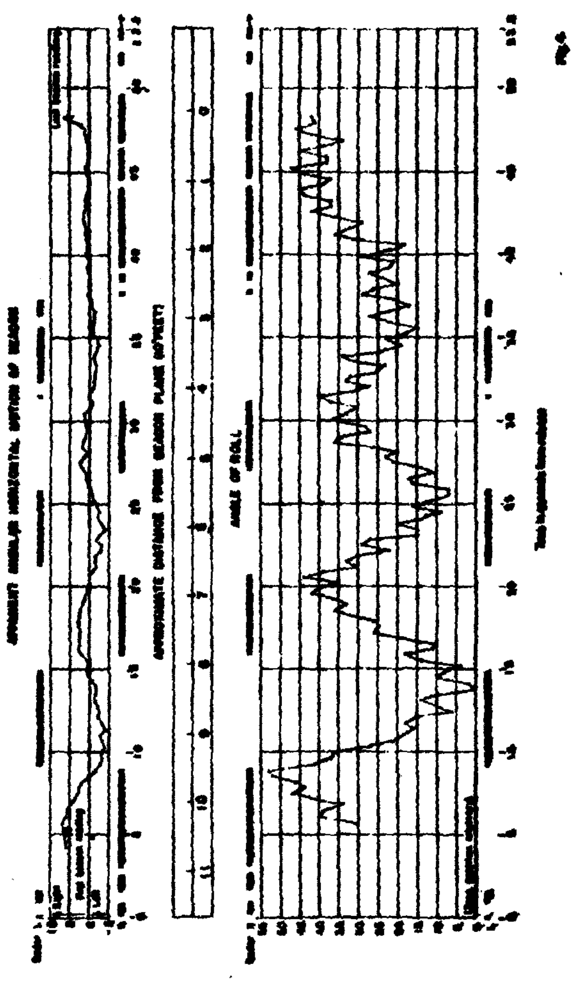

APPARENT ANGULAR HORIZONTAL MOTION OF BEACON
ANGLE OF ROLL

Fig.2





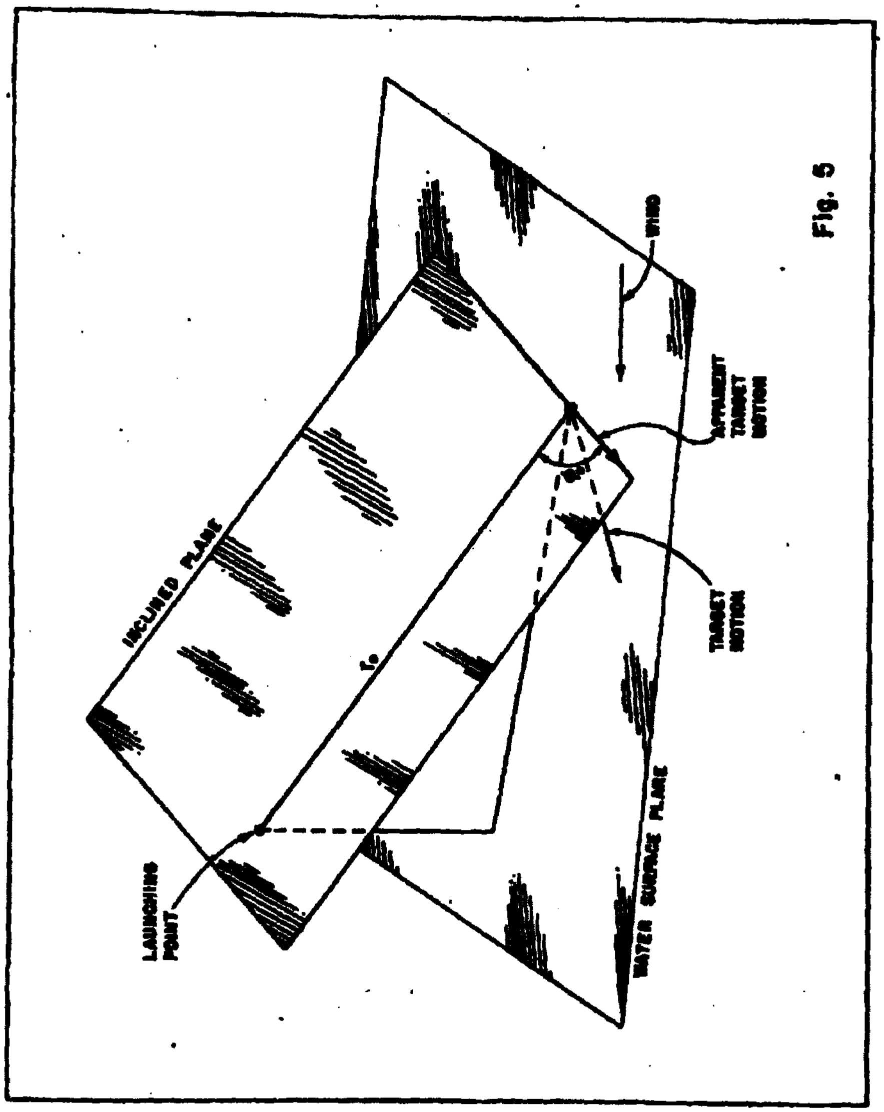
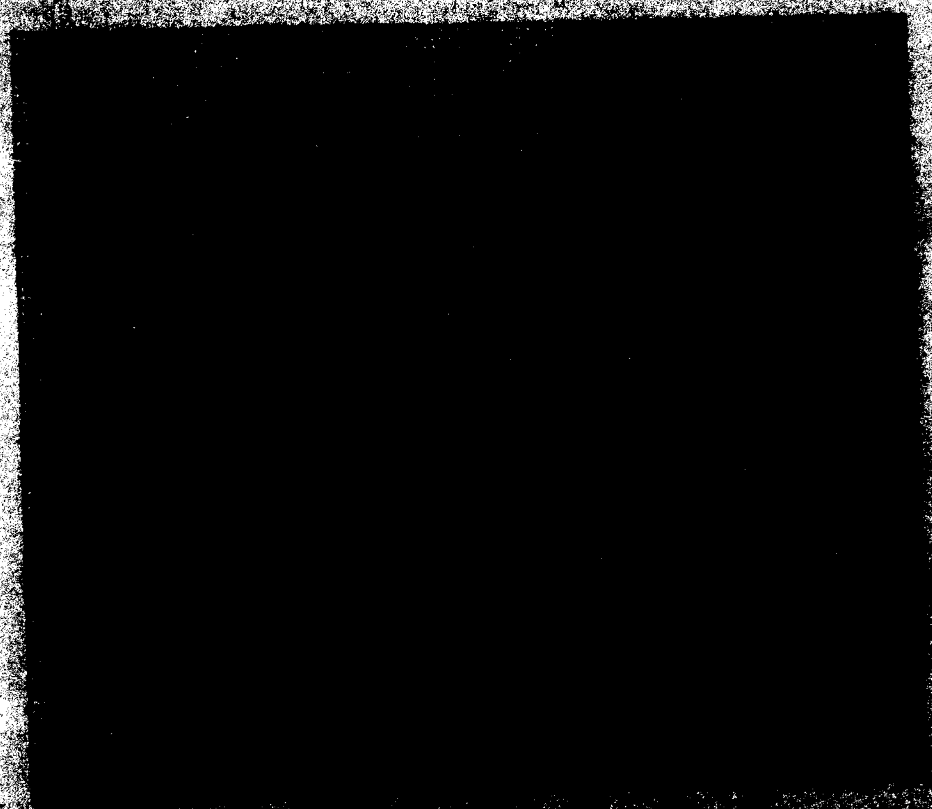

INCLINED PLANE
WATER SURFACE PLANE
LAUNCHING POINT
TARGET MOTION
APPARENT TARGET MOTION
WIND

Fig. 5



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TITLE: Some Aspects of the Design of Homing Aero-Missiles

AUTHOR(S): Dryden, Hugh L.
ORIGINATING AGENCY: National Bureau of Standards, Washington, D. C.

PUBLISHED BY: (Not published)

DASI	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
(None)	~~Confd'l~~	U.S.	Eng.	88	tables, diagrs, graphs

ABSTRACT:

The report pertains to homing devices for guided missiles. There are five classes of missiles, and they are placed in each class according to source of control. The five classes are listed and their means of control are discussed. Maneuverability of guided missiles and aerodynamic problems pertaining to them are discussed in detail. In conclusion, the various problems pertaining to launching are given.

DISTRIBUTION: Copies of this report obtainable from Air Documents Division; Attn: MCIDXD

DIVISION: Guided Missiles (1)
SECTION: Guidance and Control (1)

SUBJECT HEADINGS: Missiles - Homing guidance - Command (52100); Missiles, Guided - Control systems (33800); Missiles, Guided - Aerodynamics (82350); Missiles, Guided - Controllability (52830)

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