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RESEARCH PAPER P-146

WHISTLERS AS A LAUNCH PHASE EARLY WARNING SYSTEM (U)

L. F. Cook

D. H. Sharp

and

F. Zachariasen

February 1965

INSTITUTE FOR DEFENSE ANALYSES
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WHISTLERS AS A LAUNCH PHASE EARLY WARNING SYSTEM (U)

by

L. F. Cook*, D. H. Sharp* and F. Zachariasen**

I. Introduction

A whistler is a VLF electromagnetic disturbance that occurs in a plasma on which is imposed a magnetic field. Naturally occurring whistler signals are generated in the troposphere and propagate through the upper ionosphere along ray paths that follow rather closely the magnetic field lines of the earth. Such signals are strongly reflected in regions of rapidly changing electron densities in the ionosphere; in the case of whistlers reflection occurs from the D layer in the ionosphere and occasionally from E layer patches.

It has recently been suggested that artificially generated whistler signals may be employed as an active launch-phase early warning system.[1] The basic idea is as follows: (i) a whistler, generated by a VLF station located on an island in the Indian Ocean, travels along the magnetic field lines of the earth to the magnetic conjugate point in the USSR, (ii) there the whistler signal interacts (indirectly) with a missile while it is still in powered flight and (iii) the signal returns to the transmitter along a magnetic field

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line, carrying information about the existence of the missile in powered flight.

In order to examine such a suggestion several points must be considered. We will divide the discussion into two parts; (a) the basic properties of whistlers and (b) their possible use in detection devices.

In Section II we will discuss the properties of whistlers. In recent years it has been argued that many of the detailed observed properties of whistler propagation cannot be understood if one maintains that the ionosphere contains only smooth variations in the electron density above about 200 km. and that, consequently, columns or sheets of enhanced electron density closely aligned with the magnetic field lines of the earth must exist.[2] Such conjectured regions of enhanced electron density are called "ducts". It is by no means the case that ducts are <u>necessary</u> for the propagation of whistler signals, and there seems to be ample evidence for the existence of non-ducted whistlers.[3] However, those whistlers that do propagate via ducts exhibit a number of special properties. For example, the energy carried by such "ducted whistlers" is confined to the duct by total internal reflection at the boundaries of the ducts. Secondly, ducted whistlers should be expected to show the sporadic behavior of whistler propagation as actually observed at ground-based VLF stations.[4] It should be stressed, though, that the existence of ducts has not been directly verified by experiment.

Two possible missile launch detection systems using whistlers (as outlined above) will be considered in Section III. System I and System II are distinguished by their dependence on ducted and unducted whistler signals, respectively.

The sporadic behavior of ducted whistlers, as well as the difficulty of aiming ground-based VLF signals along ducts, leads us to the conclusion that System I is of negligible interest as an early warning sytems, or for any similar purpose.

System II, employing only unducted whistlers, is considered further and the interaction between a whistler signal and a missile in powered flight is examined. It has been shown[5] that the rocket exhaust of a missile will create an expanding region behind the missile in which the electron density is reduced relative to the ambient electron density by about an order of magnitude over a very short distance. The feasibility of the detection of missiles with whistlers would seem to reside in the possibility of detecting this thin, moving "hole" in the electron density by means of its effect on the whistler signal. Two possibilities have been considered: First, one could attempt to measure the time-delay between the signal reflected from the hole and the signal reflected from the D layer. This does not appear to be feasible, since there is generally almost total reflection from the D layer and the signal-to-noise ratio as well as the time delay is very small. Alternatively, one can make use of the fact that the hole is expanding[5] at about 1 km/sec to attempt a Doppler shift measurement. This does not appear impossible for a reasonable range of frequencies and bandwidths. Estimates of the power and sensitivity requirements relevant for this method are given in Section III.

Our conclusions are briefly summarized in Section IV. Generally speaking, we cannot argue strongly for the use of whistlers in an early warning system because the reliability of such systems appears to be quite questionable. Such a system might, however, provide useful corroborating information when used in connection with other detection methods. It is also possible that such a system could be used to monitor missile test series in the USSR, perhaps in connection with an arms freeze.

II. WHISTLERS

A. Basic Properties of Whistlers

Whistlers are electromagnetic waves of frequency around 1 - 30 kc/sec that propagate through an ionized medium in the presence of a magnetic field. From the microscopic point of view, whistler propagation takes place as a result of a collective motion of the electrons along and around the magnetic field lines, but in this discussion we will be concerned only with the macroscopic aspects of whistler propagation.

Many of the basic properties of whistlers, which depend strongly on the dispersive and anisotropic characteristics of the ionosphere, were first explained by Storey.[6] These may be summarized as follows:

(a) The whistler wave packets follow, to a certain extend, the lines of force of the earth's magnetic field.

(b) As a result of strong refraction at the base of the ionosphere and of the anisotropy of the ionosphere, wave packets entering the ionosphere from below with a wide range of incident angles will follow essentially the same path in the ionosphere as those incident vertically.

(c) The ray path followed by a whistler which enters the ionosphere with a given incident angle at a given point is independent of the frequency of the whistler (or of the central frequency in the case of a wave packet).

Many features of whistler propagation can be understood from the expression relating the index of refraction of the ionosphere to the parameters of the wave (circular frequency ω, direction of propagation specified by angle θ with respect to the direction of the magnetic line of force) and of the medium (electron density N_e, magnetic induction B), together with considerations of geometrical optics.

The dispersion relation for whistler propagation is obtained from the well-known[7] expression of Appleton and Hartree for the refractive index, n, of a monochromatic wave. Because of the relative magnitudes of the frequencies involved (see Figure I for typical values) the following inequalities are satisfied:

$$\Omega_e^2 \sin^4\theta \ll 4\omega^2(1-\gamma)^2\cos^2\theta \qquad (1a)$$

$$\Omega_e^2 \sin^2\theta \ll |2\omega^2(1-\gamma)| \qquad (1b)$$

$$|\omega| \ll |\Omega_e \cos\theta|; \quad n^2 \gg 1, \qquad (1c)$$

where

$$\Omega_e = \frac{eB}{m_e C}\,\frac{\omega}{\omega + i\nu}$$

$$\gamma = \frac{\omega_p^2}{\omega(\omega + i\nu)}$$

$$\omega_p = \left(\frac{4\pi N_e e^2}{m_e} \right)^{1/2} = \text{plasma frequency}$$

and

$$\nu = \text{collision frequency}$$

The collision frequency is much less than the wave frequency at altitudes above 100 km where most of the whistler propagation takes place, and the first two inequalities allow us to reduce the Appleton-Hartree expression to

$$n^2 = 1 - \frac{\gamma\omega}{\omega \pm \Omega_e \cos\theta} . \qquad (2)$$

When Equation (1c) is also satisfied, we have further that

$$n^2 = + \frac{\omega_p^2}{\omega\Omega_e \cos\theta} \qquad (3)$$

for the propagating wave (which is right circularly polarized).

Since we are interested in wave packets, and as we are in a dispersive and anisotropic medium, the direction of propagation of the energy, the ray direction, will differ from that of the wave propagation vector, $\vec{k}$. If we take α as the angle between these two vectors, then one finds:

$$\tan\alpha = -\frac{1}{2}\tan\theta . \qquad (4)$$

Further, the group velocity is given by

$$v_g = \frac{c}{n}\sqrt{4 + \tan^2 \theta} \; . \qquad (5)$$

Equation (4) shows that the ray vector, which is in the $(\vec{k}, \vec{B})$-plane, lies between $\vec{k}$ and $\vec{B}$.

It is of interest to know the maximum angle ψ between $\vec{v}_g$ and $\vec{B}$. A straightforward calculation leads to the result $\psi = 19°29'$, for which $\cos\theta \approx \frac{1}{\sqrt{3}}$. Thus when Eq. (3) holds, the ray vectors are confined within a cone about $\vec{B}$ with a half-angle of about 20°, and the whistler essentially follows a field line. Eq. (5), together with Eq. (3), shows that the high frequency components of the whistler packet travel faster than the low frequency components.

When a whistler is incident on the base of the ionosphere, part of the wave will be refracted up into the ionosphere, the other part will be reflected back down to earth. The qualitative description of this effect will be a subject of later discussion; here we only remark that Snell's Law applies and

$$n_o \sin \beta_o = n \sin \beta \; , \qquad (6)$$

where n_o and β_o are the index of refraction and the incident angle (with respect to the vertical) at the base of the ionosphere and n and β are the same quantities in the ionosphere. For a typical whistler, the index of refraction at an altitude of $\sim$ 100 km may be $\sim$ 30, while below the ionosphere we have $n_o \sim 1$. Consequently, whistlers incident at the base of the ionosphere at almost any angle will have their wave normals refracted into a cone within about 1° of the vertical.

Detailed numerical calculations of the paths of whistlers in the geometrical approximation have been carried out by Yabroff.[8] Although Yabroff's calculation compute the ray path starting at 300 km and thus do not deal with the problem of how the whistler enters or leaves the ionosphere, there are several interesting results:

(i) The prediction that the ray path for a given set of whistlers differing only in frequency, is the same seems to be reasonably accurate in the 5 - 20 kc/sec range.

(ii) The ray path often departs quite far from the magnetic line of force it is supposed to follow, and does not arrive at the magnetic conjugate point. This effect is strongly dependent on the magnetic latitude considered (henceforth "latitudes" will be magnetic latitudes unless the contrary is explicitly stated). At low latitudes ($\sim$25°) the ray path overshoots the conjugate point. As the initial latitude is increased the final latitude approaches the conjugate point, passes through it, and for high latitudes (~ 60°) falls short of the conjugate point. The signal arrives at the conjugate point for latitudes in the 40° to 45° range (see Fig. 2).

(iii) The angle which the whistler wave normal makes with respect to $\vec{B}$ changes with position along the path. This fact is closely related to (ii). From our previous discussion we have seen that the initial wave normals are very nearly vertical for any latitude. For low latitudes, the wave normal maintains its large angle with respect to $\vec{B}$ and the final wave normal will make a very large angle with the local vertical. As the latitude is increased the final wave normal is rotated toward the vertical, passes through the vertical, and again makes a very large angle with the vertical at high latitudes. Again the latitude from 40° to 45° in the region where the final wave normals are nearly vertical. (See Fig. 2)

(iv) The ray path and the wave normal direction follow the magnetic field line more closely if it is assumed that they are guided by ducts, i.e., by regions of enhanced electron density surrounding the magnetic field lines.

These result are of importance to us and a few comments are pertinent here. For the moment let us ignore ducts.

(a) From (i), and the existence of very strong refraction at the base of the ionosphere, it is clear that for discussions of ray paths we do not have to concern ourselves particularly with the differences between wave packet and monochromatic whistlers. (b) We see from (iii) that at most latitudes the final wave normal is not even close to being vertical. This means, because of the large change in the refractive index at the boundary of the ionosphere, that total internal reflection will generally occur and there will usually be no penetration of the signal into the troposphere. Only for latitudes in the range between 40° to 45° will there be any penetration (let us call these the magic latitudes). Note that we are here excluding the possibility of large transmission coefficients due to local irregularities in the electron density of the boundary of the ionosphere. (See § II B) (c) From (ii) we see that the reflected signal will follow a different ray path since the new initial latitude differs from the original latitude. Thus we see that once a whistler signal is generated it will bounce back and forth in the ionosphere and only very rarely penetrate into the troposphere. There will be no striking correlation in the transit times for the "bounces." (d) A special case is afforded by signals entering the ionosphere at the magic latitudes. Here the signal will bounce back and forth on the same path. Since the wave normals will always be essentially vertical at the base of the ionosphere, there will be some penetration into the troposphere. Such echoing could take place many times and definite correlations will exist in the time delays of the echoes. Such correlations in fact exist in nature[2], and it was precisely because of such structure that ducts were introduced into the study of whistlers. Point (iv) above shows that ducts will yield the same kind of correlation as those present at the magic latitudes, but at a variety of latitudes and therefore a variety of ray paths. It very well may be that ducts are necessary to understand some important details of whistler phenomena, but conceivably some of the experimental data can be understood in terms of these magic latitudes. In any event, a discussion of the pros and cons of ducts and magic latitudes is out of place here. No one doubts the <u>existence</u> of unducted whistlers, and it is these that will form the basis of a possible early warning system. An early warning system based on ducted whistlers will be shown not to be feasible.

B. Reflection and Transmission of Whistler Signals at the Boundary of the Ionosphere

The use of whistlers as an early warning system of course depends to a high degree on the power and sensitivity requirements of the transmitter detector system which in turn depends on the energy transmission and reflection at ionospheric boundaries. It is evident that a quantitative discussion of this rather complicated wave propagation problem would require a numerical solution of Maxwell's equations. Such a discussion may also involve a more detailed knowledge of the electron density profile than is presently available. However, it is possible to give a crude discussion of this problem which is adequate for our purpose. It is not expected that these results will come as a surprise to experts, but we present them as a basis for the remarks in Section III. We find, in brief, that the reflection and transmission coefficients for whistler signals are rather sensitive to local irregularities in the electron density profile.

We consider an atmosphere, plane-stratified in the z-direction. It would of course be incorrect to use a plane geometry if we were to calculate the paths of whistlers, but it is reasonable for the calculations of reflection and transmission at the boundary of the ionosphere. We suppose that the electromagnetic wave is linearly polarized with its electric vector perpendicular to the plane of incidence ($y - z$ plane) and with a time dependence $e^{i\omega t}$ (TE wave). Following Born and Wolf[9], for example, one finds for the only non-zero component of the electric field, E_x:

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + n^2 k_o^2 E_x = \frac{d}{dz}(\log \mu)\,\frac{\partial E_x}{\partial z}\,, \qquad (1)$$

where $n^2 = \epsilon\mu$, $k_o = \omega/c = 2\pi/\lambda_o$, μ is the magnetic permeability ($= 1$ in our case) and ϵ is the dielectric constant. If we assume that the solution has the separable form $E_x(y, z) = Y(y)U(z)$, then one finds

$$Y(y) = Ae^{ik_o\alpha y}\,; \quad \alpha = n \sin\theta\ .$$

For $U(z)$ we find, with $\mu = 1$ and $y = 0$ (for convenience), the equation:

$$\frac{d^2U}{dz^2} + K^2(z)U = 0\ , \qquad (2)$$

with $K(z) = k_o n \cos\theta$. The components of the magnetic field can be expressed[9] in terms of $U(z)$, and the TM wave can be treated in a similar way.

Now let us turn to Eq. (2), and consider first the part of the whistler path at altitudes well above the reflection region, say above 300 km. Here the W.K.B. approximation can be shown to apply.[10] As a result we can see, recalling that the correct expressions should actually be written in spherical coordinates, that the whistler wave in going from 300 km up along a magnetic line of force suffers a phase change, together with an amplitude modulation which changes slowly with position. This solution includes the results of the ray theory.[8]

At each end of the path, the whistler goes through a region in which the W.K.B. solution fails badly. In this region, therefore, we will solve Eq. (2) directly. First we consider a whistler going up into the ionosphere, so we consider a transition from a rare to a dense medium. Also we consider only normal incidence and we will correct the results for general incident angles later.

We consider three regions (see Fig. 3). For $z < 0$ we assume that $k(z) = k_o$, the free space value. For $z > 0$ we set $k(z) = k = n\nu/c$. In between we assume the form,

$$k(z) = \frac{k_o}{\left[1 - \left(1 - \frac{k_o}{k}\right)\frac{z}{D}\right]} . \qquad (3)$$

Our final solutions in these three regions are as follows:

$$U = e^{ik_o z} + Re^{-ik_o z} \qquad z < 0$$

$$U = [1 - (1 - \frac{k_o}{k}) \frac{z}{D}]^{\alpha} \qquad 0 < z < D \qquad (4)$$

$$U = Te^{ikz} \qquad D < z$$

One finds for the parameter α;

$$\alpha_{\pm} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \Delta^2}\,, \qquad (5)$$

$$\Delta = \frac{k_o D}{1 - k_o/k}\,.$$

The general solution for U in the region $0 < z < D$ is thus,

$$U(z) = A[1 - (1 - \frac{k_o}{k}) \frac{z}{D}]^{\alpha_+} + B[1 - (1 - \frac{k_o}{k}) \frac{z}{D}]^{\alpha_-}\,. \qquad (6)$$

At the boundaries $z = 0, D$ we require continuity of U and dU/dz. This yields four algebraic equations which may be solved for A, B, R and T. In particular we find

$$R = \frac{1}{(\frac{1-Q_+}{1+Q_+}) - (\frac{k_o}{k})^{\sqrt{1-4\Delta^2}} (\frac{1-Q_-}{1+Q_-})} + \frac{1}{(\frac{1-Q_-}{1+Q_-}) - (\frac{k}{k_o})^{\sqrt{1-4\Delta^2}} (\frac{1-Q_+}{1+Q_+})}\,, \qquad (7)$$

and

$$T = e^{-ikD}\left(\frac{k_o}{k}\right)^{\alpha_+}\left(\frac{1-Q_+}{1+Q_+}\right)\frac{1}{\left(\frac{1-Q_+}{1-Q_+}\right) - \left(\frac{k_o}{k}\right)^{\sqrt{1-4\Delta^2}}\left(\frac{1-Q_-}{1+Q_-}\right)}\,,$$

$$+\, e^{-ikD}\left(\frac{k_o}{k}\right)^{\alpha_-}\left(\frac{1-Q_-}{1+Q_-}\right)\frac{1}{\left(\frac{1-Q_-}{1+Q_+}\right) - \left(\frac{k}{k_o}\right)^{\sqrt{1-4\Delta^2}}\left(\frac{1-Q_+}{1+Q_+}\right)}\,, \qquad (8)$$

where $Q_{\pm} = \alpha_{\pm}/i\Delta$.

One can easily check these equations in various limiting cases. For example, when $D \longrightarrow 0$, $\Delta \longrightarrow 0$ we have

$$R \longrightarrow \left(\frac{n/n_o-1}{n/n_o+1}\right)$$
$$T \longrightarrow 2/(n/n_o+1)\,. \qquad (9a)$$

As $k \longrightarrow k_o$, $\Delta \longrightarrow \infty$ and we have

$$R \longrightarrow 0, \quad T \longrightarrow e^{ik_oD}. \qquad (9b)$$

Finally, if $k/k_o \longrightarrow \infty$ and $D \longrightarrow 0$, we find $R \longrightarrow -1$, $T \longrightarrow 0$.

One can include the effect of non-vertical incidence of the wave by replacing Δ in Eq. (5) by

$$\Delta = k_oD \cos\beta_o/\left(1 - \frac{k_o}{k}\,\frac{\cos\beta_o}{\cos\beta}\right), \qquad (10)$$

where β_o and β are the initial and final angles, respectively. We can express R in the somewhat more convenient form.

$$R = \frac{1}{2i\Lambda - x\,\frac{[1+v^x]}{[1-v^x]}}\,, \qquad (11)$$

where $x = \sqrt{1 - 4\Delta^2}$, $v = k_o \cos \beta_o / k \cos \beta$. A rough plot of $|R|^2$ against Λ for a ratio of wavelengths $\lambda_o/\lambda \sim 55$ is given in Fig. 3.

We see from Eq. (9a) that at a sharp boundary, $\Lambda \longrightarrow 0$, the whistler signal is almost totally reflected. However, as Fig. 4 shows, one can pass from a region of almost total reflection to one of almost total transmission as Λ varies from approximately zero to one or two. There are several sources of variations of Λ, and it is not at all improbable that they can change Λ to such an extent that marked changes in the reflection and transmission coefficients will occur from one local region to another. Two obvious ways of changing Λ are:

i) Changes in the frequency of the wave (changing k_o).

ii) Changes in the angle of incidence (β_o).

However the most important considerations involve:

iii) Local changes in the electron density profile.

There is ample evidence[11] for a day-night variation in the electron density. It does not seem implausible on the basis of this evidence to suppose that the distance D over which the index of refraction can change from 1 to 30, which we here nominally take as about 20 km, can increase by a factor of two. If $\Delta \sim 1/2$, for example, then an increase of D by a factor of two can take us from a region of almost total reflection to almost total transmission (see Fig. 4).

There is also experimental evidence for <u>horizontal</u> irregularities in the electron density.[12] Such irregularities could result in whistlers penetrating the ionosphere at different points.

We remark that the possible importance of local irregularities in the electron density on the propagation of whistlers has been discussed by Budden.[13] We regard the above as only a slight amplification of his remarks.

So far we have considered the entry of a whistler into the ionosphere, a transition from a rare to dense medium. It is of course necessary to treat the opposite case of a transition from a dense to a rare medium. The reflection and transmission coefficients for this case can be obtained from the above results by replacing Δ in Eq. (10) by

$$\Lambda = \frac{k\, D \cos\beta}{1 - \dfrac{k \cos\beta}{k_o \cos\beta_o}} \,. \qquad (12)$$

The effect of local irregularities, etc. on the reflection and transmission of the whistler signal will be equally marked in this situation.

III. WHISTLERS AS A LAUNCH PHASE EARLY WARNING SYSTEM

In this section we remark briefly on a possible missile detection system, utilizing an active VLF source and receiving Doppler-shifted reflected signals from a region of reduced electron density in the wake of a missile penetrating the F_2 layer.

As indicated in the Introduction, two alternative systems may be contemplated. The first, System I, is based on ducted whistlers. We do not believe that such a system is feasible. In dismissing such systems, we have been guided by the facts that (i) there is general agreement that unducted whistler propagation is a common phenomenon, (ii) the ducted whistlers often show a very sporadic behavior and finally (iii) if ducted whistler propagation in the ionosphere is considered, then the signal would not be expected to enter the ionosphere from the earth-ionosphere wave guide until it arrived at the end of a duct, so that "aiming" at a missile site located at the magnetic conjugate point to the VLF transmitter becomes difficult, though perhaps not impossible since one only has to aim with an accuracy of perhaps 1000 miles.

Even in assessing the value of System II, based on unducted whistlers, it is important to keep several large uncertainties in mind. First, it may be that penetration of both ducted and unducted whistlers into the ionosphere requires irregularities in the electron density of the lower layers, which exist only at isolated points in space, thus reintroducing the "aiming" problem mentioned above for ducted whistlers. Second, even if the VLF signal can penetrate the ionosphere directly over the transmitter, the experimental erraticism of detectable reflected whistlers makes it unlikely that one could expect the system to function with 100% reliability. Third, there are considerable uncertainties in the estimates of the strength of the expected reflected signal partly because, in the absence of ducts, it is difficult to be sure of how much divergence of the VLF beam one may expect in the ionosphere and partly because transmission into and out of the ionosphere depends sensitively on the detailed shape of the electron density profile. Fourth, there might on rare occasions be natural irregularities in the ionosphere which produce Doppler shifts comparable to the one expected from a missile, thus leading to false signals.

In view of all this, it seems unlikely that complete reliance could be placed on such a detection or early warning system, even if it more or less works, but it may perhaps still be of some interest if used in conjunction with other detection schemes.

In the following analysis, we shall assume ducts to be unnecessary and that penetration of the ionosphere at any desired point is always possible.

As an ascending missile passes through the region of maximum electron density, at about 300 km altitude, one may expect that there is formed behind it an expanding hole,[5] of decreased electron density. Roughly, this hole will grow at a velocity of about 1 km/sec until it reaches a size of about 10 km. The electron density may be expected to be reduced to about a factor of 10 below ambient, and the distance in which this decrease occurs is fairly small -- perhaps less than 1 km.[5]

Qualitatively, then, one could envisage the following system: A VLF transmitter is located at the conjugate point to the rising missile. Signals are emitted, penetrate the ionosphere, propagate in the whistler mode roughly along the field line, reflect off the hole in the electron density caused by the missile, return along the field line (Doppler shifted by the expanding wall of the hole) and re-penetrate out of the ionosphere to the receiver located near the transmitter.

Let us try to examine the various stages of this process in somewhat more detail.

We shall first estimate the expected returned power from the missile on the basis of a very crude model which should give a lower bound to the power returned. Let us follow the path of a given ray as indicated in Fig. 5. If it leaves the transmitter at an angle θ to the vertical, and if we take the refractive index to be a constant n in the ionosphere and $n_o = 1$ below it, then the ray, after penetrating the ionosphere, is at an angle

$$\frac{n_o}{n}\,\theta$$

to the vertical. The transmission coefficient into the ionosphere is then approximately

$$\left(\frac{2}{\frac{n}{n_o} + 1}\right)\sqrt{\frac{n}{n_o}} \quad .$$

The total lateral distance away from the transmitter when the ray reaches the hole behind the missile is then

$$h\theta + \frac{n_o}{n}\,\theta L \quad ,$$

where h is the height of the ionosphere and L is the path length through to the conjugate point. (In contrast to the return signal, after reflection, we do not assume here that the beam spreads over the usual 20° half angle whistler cone, since the wave normals after transmission into the ionosphere are so well clustered around the vertical.) The maximum allowed angle for seeing the hole is then

$$\theta_{max} = R/\left(h + \frac{n_o}{\bar{n}} L \right) \quad ,$$

where R is the radius of the hole, and for simplicity we assume the hole exactly "above" the transmitter and that the earths magnetic field is vertical. After reflection from the hole (which we take to have a constant index of refraction $\bar{n}$) with reflection coefficient

$$\frac{\frac{n}{\bar{n}} - 1}{\frac{n}{\bar{n}} + 1}$$

we may assume the wave normals in the reflected beam to be distributed roughly uniformly over an entire hemisphere. The reflected rays then propagate back along the field line, but spread over a 20° expanding cone, and they therefore arrive at the surface of the ionosphere above the transmitter spread out over an area of radius about $L \tan 20^\circ$. Now all wave normals outside an acceptance cone of half angle n_0/n will be totally reflected. Inside this cone, the wave will be transmitted with a transmission coefficient

$$\left(\frac{2}{\frac{n_0}{n} + 1}\right)\sqrt{\frac{n_0}{n}} \quad .$$

We may put all this together, assuming the transmitted power is radiated nearly isotropically. We then find the returned power per unit area to be the following:

$$\bar{P} = P_0 \cdot \frac{1}{2}\left(\frac{R}{h + \frac{n_0}{n} L}\right)^2 \cdot \left(\frac{2}{\frac{n}{n_0} + 1}\right)^2 \left(\frac{n}{n_0}\right)\left(\frac{\frac{n}{\bar{n}} - 1}{\frac{n}{\bar{n}} + 1}\right)^2$$

$$\cdot \frac{1}{2}\left(\frac{n_0}{n}\right)^2 \cdot \left(\frac{2}{\frac{n_0}{n} + 1}\right)^2 \left(\frac{n_0}{n}\right) \frac{1}{\pi L^2 \tan^2 20^\circ} \quad .$$

If we take $n \approx 30$, which corresponds to an electron density of 10^5 and a frequency of 5 kc, then $\bar{n} \approx 10$. We may assume $h = 100$ km, $L = 2 \times 10^4$ km, and $R = 10$ km. Then

$$\bar{P} \approx 10^{-24} P_o/(\text{meter})^2 \quad .$$

This number is quite small. If we suppose that the transmitter can put out 1 megawatt at 5 kc (a non-trivial job) we find $\bar{P} \sim 10^{-18}$ watts/m^2, which may be marginally visible with a very narrow bandwidth receiver. If we go to a higher frequency, so that the transmission coefficients into and out of the ionosphere increase, these values improve somewhat. At 25 kc, for example, we may assume $n = 13.5$, $\bar{n} = 4.5$ and thus find

$$\bar{P} = 6 \times 10^{-24} P_o/(\text{meter})^2 \quad .$$

However, at 25 kc it is easier to lose the whistler mode of propagation at high latitudes.

The model used here is unrealistic in that the actual index of refraction is not discontinuous as one enters the ionosphere. Any smoothing out of our step function profile will increase the transmission into the ionosphere, perhaps considerably, since the transmission coefficient is a rather sensitive function of the ratio of the wavelength to the distance over which the electron density changes appreciably (see Section II b). In general, higher frequencies should penetrate more easily.

One could perhaps have the option of using several different frequencies, between which one could switch according to how well a particular frequency is getting through. The ability to transmit could be continuously monitored by sending out short pulses and listening for the signal reflected from the other end of the ionosphere. The transmission could consist therefore of a mixture of such short ($\ll$ 1 sec) monitoring pulses and long ($>$ 1 sec) pulses for Doppler measurements, as described below.

Perhaps some improvement may also come about if a slightly directional transmitter could be used, though with a 60 km free space wavelength this seems unlikely. Again a higher frequency would be desirable; at 25 kc the wavelength is only 12 km.

Summarizing then one may hope to have a sensitivity in the receiver of perhaps 3×10^{-18} watts/m^2 with a signal to noise ratio of 10. Our 10^{-18} watts/m^2 estimate for the returned signal at 5 kc is thus not visible, without some enhancement in transmission due to a smoother variation of electron density. At 25 kc, however, the signal would be barely visible.

Let us next turn to the Doppler shift to be expected. If v is the velocity of the surface of the hole, along the ray direction, then the Doppler shift is

$$\Delta\omega = \frac{2\omega n v}{c} .$$

If we consider $v = 1$ km/sec(5) to be the maximum value of v, we have

$$\frac{\Delta\omega}{2\pi} = 3 \text{ cps at } 5 \text{ kc} .$$

Here we have taken $n \approx 100$, which corresponds to 10^6 electrons/cc at 5 kc, which is a reasonable F_2 layer value.(11) Since n decreases with increasing frequency, $\Delta\omega$ varies slowly with ω. For $\omega/2\pi = 25$ kc, for example,

$$\frac{\Delta\omega}{2\pi} \sim 6 \text{ cps} .$$

Since the component of velocity in the desired direction changes from 0 to 1 km/sec as we vary over the surface of the hole, the return will contain shifts from 0 to the values given above.

In order to distinguish this signal from the background produced by the signal returned by reflection at the other end of the ionosphere, the receiver should have a bandwidth of no more than about 1 cycle.

It is important to know what natural Doppler shifts we may expect, due perhaps to motion of irregularities in ionosphere over the conjugate point to the transmitter. There seems to be little information available on this point. Estimates however for the speed at which irregularities in the ionosphere move range up to only 100 or 200 meters/sec, so probably this background effect will not be severe.

Geographically, most of the interesting conjugate points to the USSR lie in the Southern Indian Ocean (see Fig. 6). Parts of Western Australia may be of some value as observation points, but otherwise one is confined to only a few islands. Of these, all are French except for Heard Island (Australian) which is entirely covered with ice. It may therefore be necessary to think in terms of a ship-based system which will, of course, hardly lessen the difficulties.

ACKNOWLEDGEMENTS

We wish to thank Drs. D. O. Caldwell and A. Peterson for a number of helpful discussions.

(Conclusions follow)

IV. CONCLUSIONS

1. Systems relying on ducted whistlers are of negligible value.

2. Systems relying on non-ducted whistlers are of marginal feasibility because:

   a. Power and sensitivity requirements seem, according to our estimates, to be at the limit of what is presently possible.

   b. Erratic operation must be expected because of fluctuations in the electron density in the D layer.

   c. The geographical limitations are severe.

   d. The model on which our numerical estimates are based is very crude, and may possibly be misleading.

3. Even assuming that our numerical estimates are pessimistic we find it difficult to argue that whistlers could form the basis of a reliable early warning system.

4. However, there may be other applications of a whistler system, possibly in connection with monitoring a missile test freeze, but to assess the value of any such use more detailed study would be required.

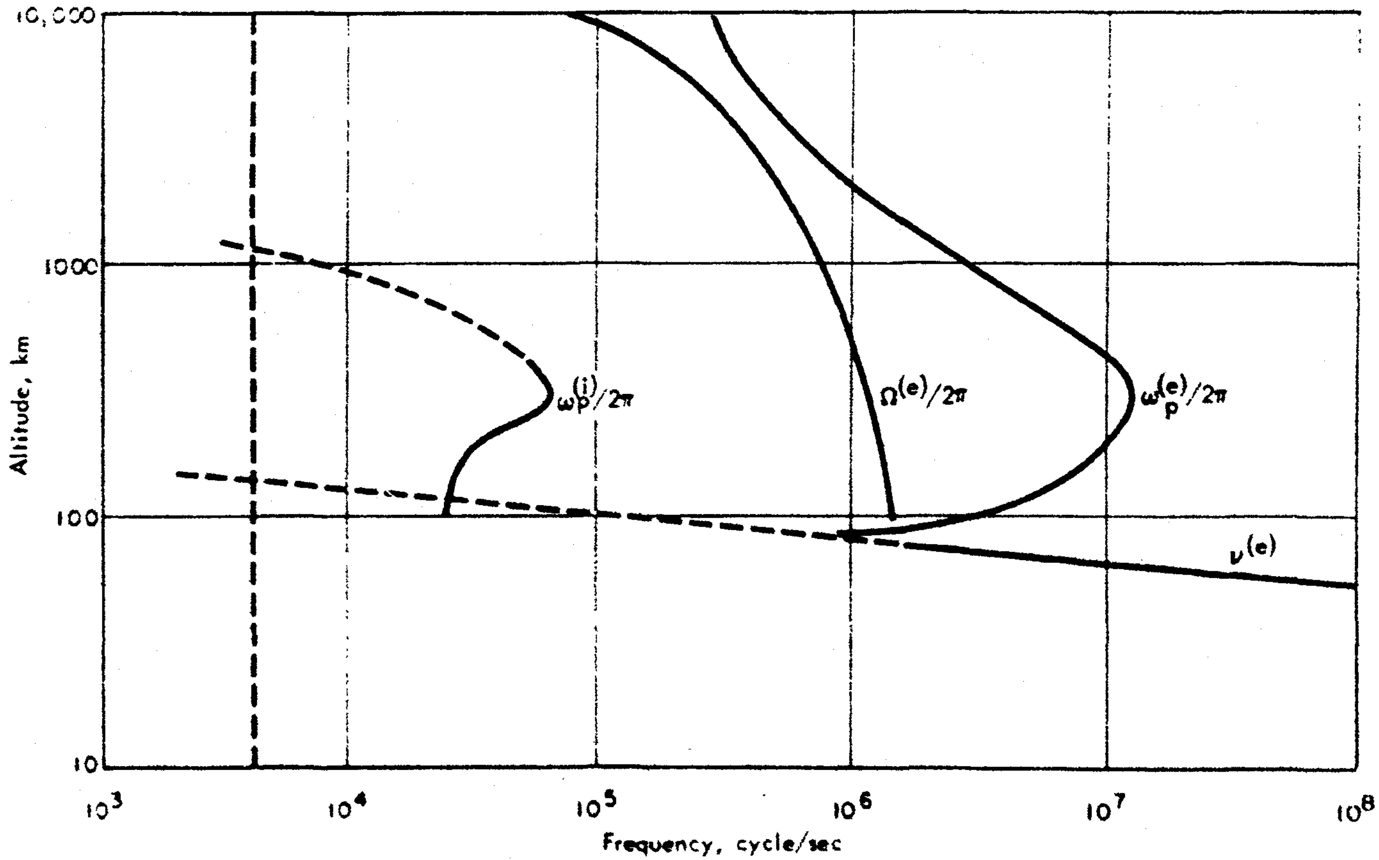

FIGURE 1 Atmospheric Parameters

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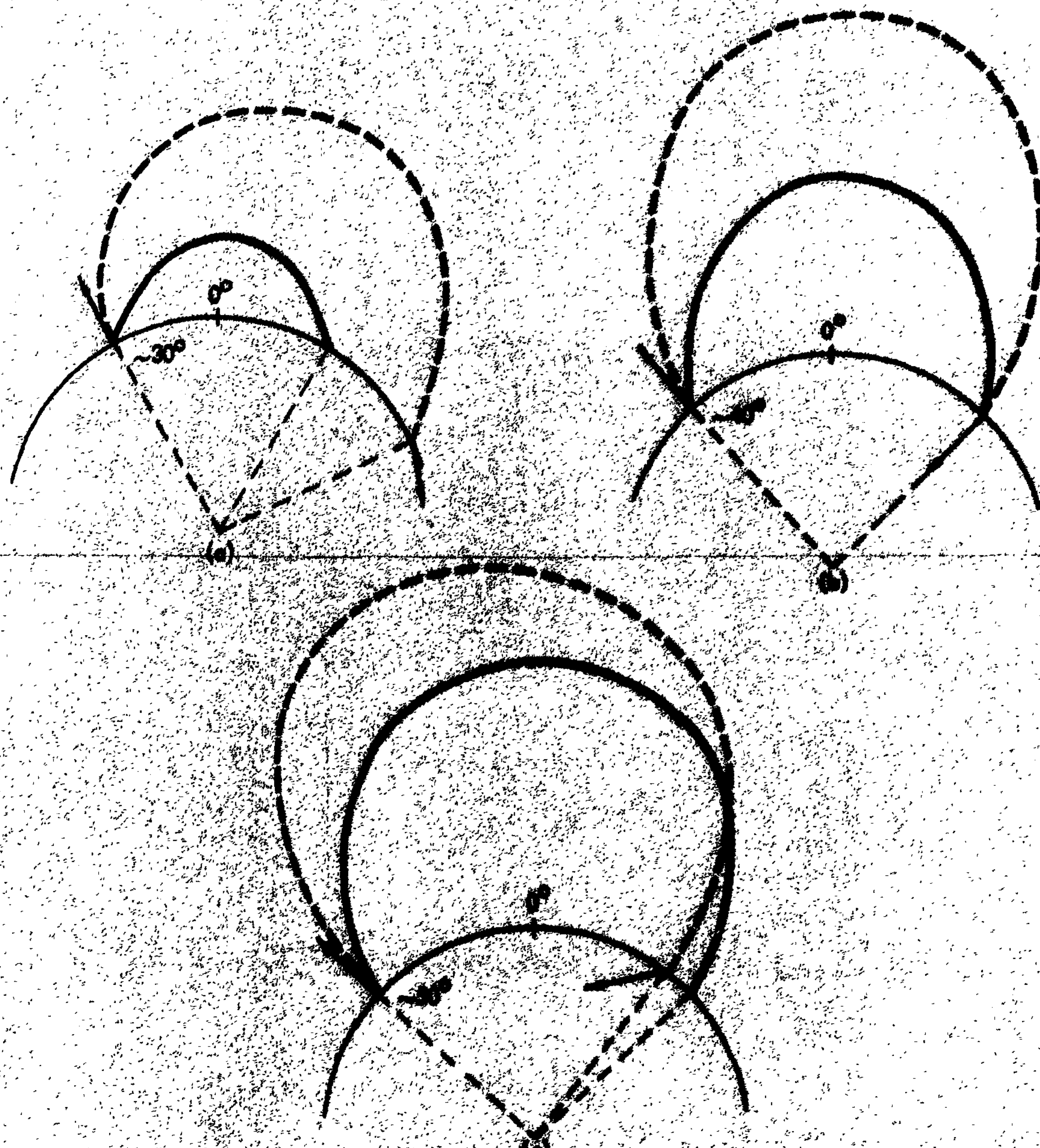


FIGURE 2 Whistler Ray Paths (Dotted Paths) Beginning at Magnetic Latitudes of about 30°, 40°, and 50°

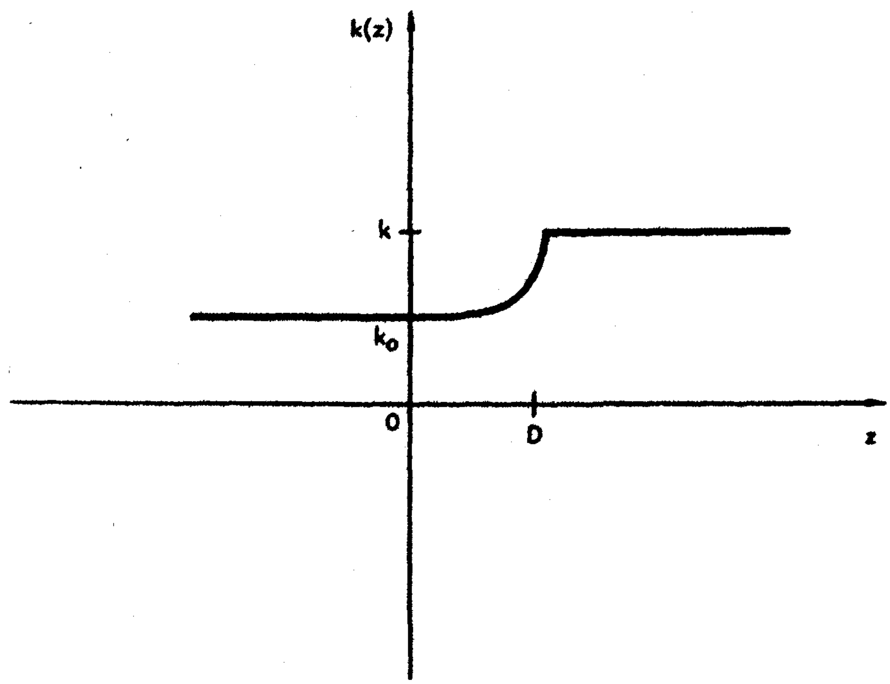


FIGURE 3 Assumed Form for k(z) vs. z at Boundary of the Ionosphere

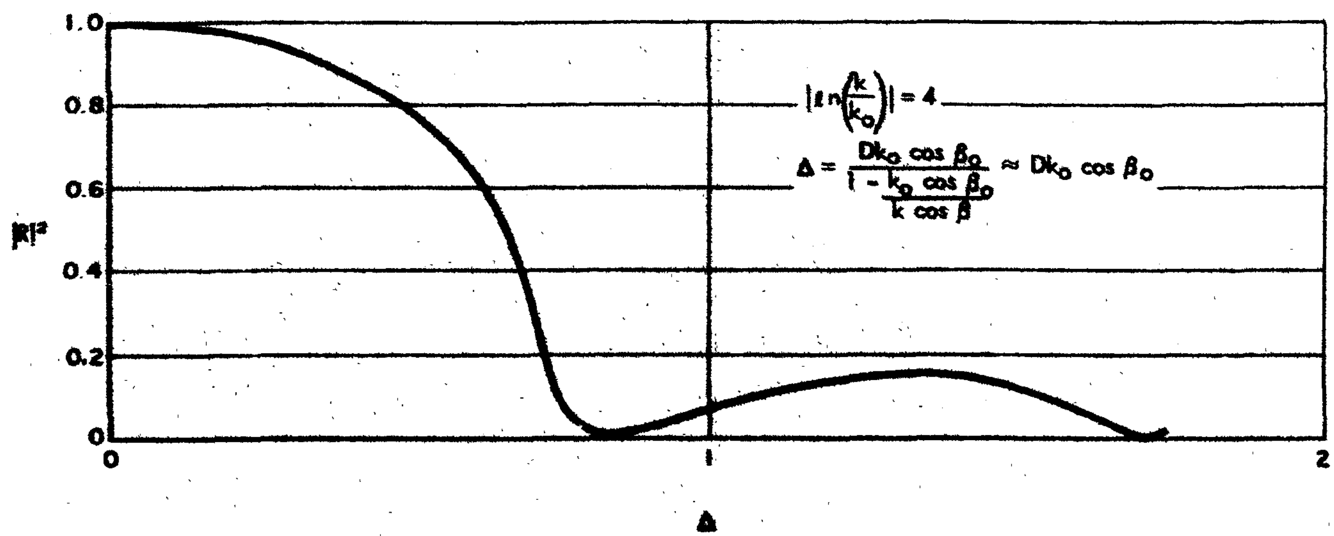


FIGURE 4 Plot of $|R|^2$ vs. Δ with $\lambda_o/\lambda \approx 55$

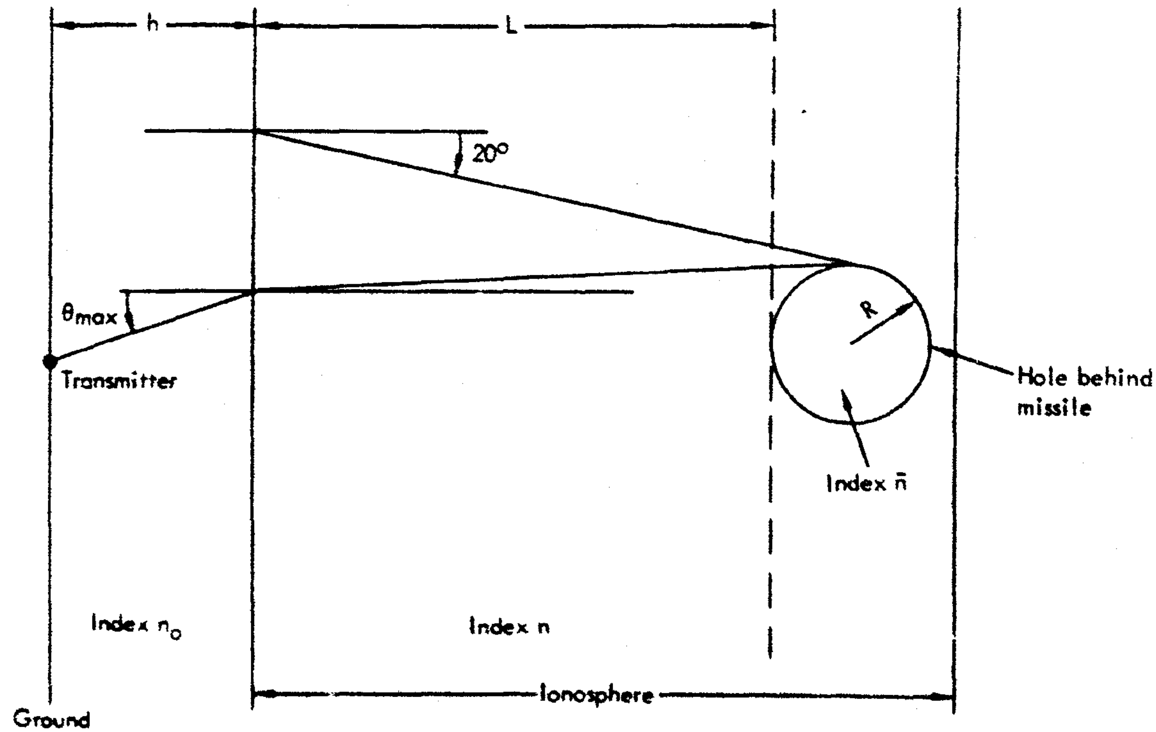
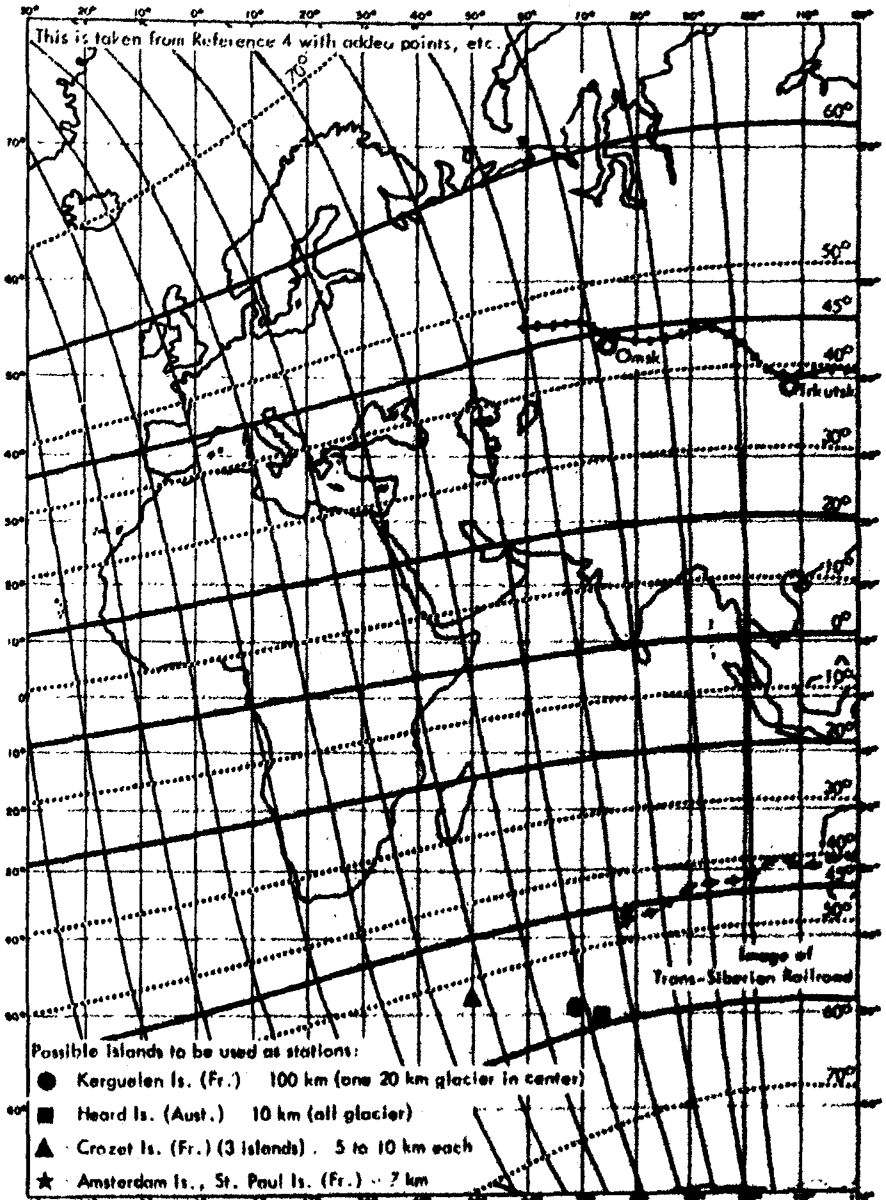


FIGURE 5 Schematic Diagram of System II



The magnetic image of the Trans-Siberian Railroad is also shown.

FIGURE 6 Map of VLF Transmitters, Field Stations, and Approximate Vestine Conjugates

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